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Procedures and Computer Programs for Reduction of Ballistic Plates from Trailblazer I and II Programs

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LINCOLN LABORATORY

PROCEDURES AND COMPUTER PROGRAMS FOR REDUCTION OF BALLISTIC PLATES FROM TRAILBLAZER I AND II PROGRAMS

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Group 21

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ABSTRACT

The Optical Reduction Programs have been written to obtain the space trajectory of a re-entering body from measurements of position on two photographic plates taken of a re-entry event from two separate stations. The method of analysis follows closely that developed by Whipple and Jacchia[1]. The Programs are a modification for the Lincoln Laboratory 7094 computer of a program originally written at the Harvard Observatory for reduction of meteor trails. The computation is divided into two programs:

1. The Plate Calibration Program calibrates the plate by using coordinates and plate measurements of known stars as reference points.

2. The Optical Trajectory Program computes range, height, time, direction cosines, and distance along the trail from the measured points on the trail.

This paper discusses these two programs in detail, including program listings, flow charts, and directions for running the programs. The mathematical background and the experimental method for obtaining the input data are also discussed.

Accepted for the Air Force
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GLOSSARY

Sky and Plate Coordinates

Symbol	Definition
a, A	Altitude and azimuth as referred to the local horizontal
δ, H	Declination and hour angle as referred to the equator
δ, α	Declination and right ascension as referred to the equator
β, λ	Celestial latitude and longitude as referred to the ecliptic
ξ, η, ζ	Direction cosines in the altitude-azimuth system
l, m, n	Direction cosines in the declination-hour angle system
λ, μ, ν	Direction cosines in the declination-right ascension system
τ	Vernal equinox or first point of Aries
ϵ	Obliquity of the ecliptic (23 1/2°)
$\bar{\xi}$, $\bar{\eta}$	Standard rectangular plate coordinates with origin at the optical plate center
X, Y	Measured rectilinear coordinates on the photographic plate
σ	Angular distance from the optical plate center
ΔX, ΔY	Corrections to measured rectilinear coordinates on the photographic plate
X_x, X_y, X_z Y_x, Y_y, Y_z Z_x, Z_y, Z_z	Precession direction cosines for the declination - right ascension system

u_α, u_δ	Proper motion in right ascension and declination per year
a,b,c,d	Plate constants in four constant solution
A,B,C,D	Inverse plate constants in four constant solution
$a_x, a_y, a_z, b_x, b_y, b_z$	Plate constants in six constant solution
$a_\xi, a_\eta, b_\xi, b_\eta, c_\xi, c_\eta$	Inverse plate constants in the six constant solution

Time and Terrestrial Coordinates

S.T. or θ	Sidereal time at observer
θ_G	Sidereal time at Greenwich
θ_o	Sidereal time at 0000 Greenwich time at Greenwich
$\Delta\theta$	Correction to sidereal time required since a solar day is longer than a sidereal day
U.T.	Universal or Greenwich time
L	Terrestrial longitude
ϕ	Terrestrial latitude (geographic)
ϕ'	Terrestrial latitude (geocentric)
ρ	Radius of earth at sea level at observer's latitude
R	Radius of earth at observer
ξ_{AB}, η_{AB}, ζ_{AB}	Direction cosines in equatorial system of the vector from station A to station B
$\xi_{AB_o}, \eta_{AB_o}, \zeta_{AB_o}$	Direction cosines in the equatorial system of the vector from station A to station B at sidereal time equal to zero at station A
λ_z, μ_z, ν_z	Direction cosines in the equatorial system of the zenithal point of the observer

Other Symbols

f	Lens focal length
l	Length of the trail in angular measure
Q	Angle between the two meteor trails seen from the two stations
S_A	Distance from station A to the plane determined by the re-entry object and station B
R_{Ai}	Range from station A to the point i on the re-entry object
H	Height above mean sea level
D	Distance along the trail from an arbitrary zero point
P	Period of revolution of shutter
N	Number of occultations of shutter per revolution
t	Relative time from arbitrary zero time
ω	Angle that blade edge makes with dash

Subscripts

s	Coordinates of stars
c	Coordinates of plate center
q	Center of rotating shutter

Double letter subscripts are used for points along the trail. The first letter is always A (for station A) or B (for station B)

i	A point on the trail - any shutter break or segment
b	Beginning of photographic trail
e	End of photographic trail

p	Pole of trail considered as a part of great circle
G	Greenwich
ξ, η	Refer to $\bar{\xi}$ and $\bar{\eta}$ standard plate axes
X, Y	Refer to X and Y axes on plate
R	Radiant of the event

Plate Calibration Program

Computer Symbol	Math Symbol	Definition
TITLE	--	Title of event - any 72 characters desired
NN	--	Number of sets of precession constants
N	n	Number of stars
XX,XY,XZ	X_x, X_y, X_z	
YX,YY,YZ	Y_x, Y_y, Y_z	Precession constants
ZX,ZY,ZZ	Z_x, Z_y, Z_z	
ACHR,ACMIN,ACSEC	α_c	Right ascension of plate center in hours, minutes, seconds
DCDEG,DCMIN,DCSEC	δ_c	Declination of plate center in degrees, minutes, seconds
XC,YC	X_c, Y_c	Measured coordinates of plate center
M	--	Number of precession set to use with plate center or star
DTH	$\Delta\theta$	Correction in sidereal time to be used with non-tracking cameras
AC	α_c	Right ascension of plate center in radians
DC	δ_c	Declination of plate center in radians

ELC,EMC,ENC	λ_c, μ_c, ν_c	Direction cosines of unprecessed plate center in declination right ascension system
ELCP,EMCP,ENCP	λ_c, μ_c, ν_c	Direction cosines of precessed plate center in declination right ascension system
ELETA,EMETA,ENETA	$\lambda_\eta, \mu_\eta, \nu_\eta$	Direction cosines of $\overline{\eta}$ axis
ELEXI,EMEXI	λ_ξ, μ_ξ	Direction cosines of $\overline{\xi}$ axis
AHR,AMIN,ASEC	α_s	Right ascension of star in hours, minutes, seconds
A	α_s	Right ascension of star in radians
DDEG,DMIN,DSEC	δ_s	Declination of star in degrees, minutes, seconds
D	δ_s	Declination of star in radians
X, Y	X, Y	Measured coordinates of star
ELS,EMS,ENS	λ_s, μ_s, ν_s	Unprecessed direction cosines of star in declination-right ascension system
EL,EM,EN	λ_s, μ_s, ν_s	Precessed direction cosines of star in declination-right ascension system
COSSIG	σ_s	Angular distance of star from plate center
EXI,ETA	$\overline{\xi}$, $\overline{\eta}$	Standard plate coordinates of star
A,B,C,D	a,b,c,d	Plate constants from four constant solution
ACAP,BCAP,CCAP,DCAP,ECAP	A,B,C,D,-A	Inverse plate constants from four constant solution
DX,DY	ΔX, ΔY	Residuals in X and Y
CF,SF	$\cos \Delta\theta$, $\sin \Delta\theta$	Cosine and sine of $\Delta\theta$

ELC,EMC,ENC	$\lambda_c{}', \mu_c{}', \nu_c{}'$	Corrected (for $\Delta\theta$) direction cosines of the plate center and the standard axes in the declination-right ascension system
ELEX,EMEX,ENEX	$\lambda_\xi{}', \mu_\xi{}', \nu_\xi{}'$	
ELET,EMET,ENET	$\lambda_\eta{}', \mu_\eta{}', \nu_\eta{}'$	

SIXCON

SEXI2	$\Sigma \bar{\xi}^2$	
SEXIET	$\Sigma \bar{\xi}\,\bar{\eta}$	
SEXI	$\Sigma \bar{\xi}$	
SEXEXI	$\Sigma x \bar{\xi}$	
SETA2	$\Sigma \bar{\eta}^2$	
SETA	$\Sigma \bar{\eta}$	
SEXETA	$\Sigma x \bar{\eta}$	
SEX	Σx	
SEYEXI	$\Sigma y \bar{\xi}$	
SEYETA	$\Sigma y \bar{\eta}$	
SEY	Σy	
A		Coefficient matrix
B		
ASEXI,BSEXI,CSEXI	a_ξ, b_ξ, c_ξ	Inverse plate constants for six constant solution
ASETA,BSETA,CSETA	a_η, b_η, c_η	

Optical Trajectory Program

TITLE	--	Title of event - any 72 characters desired
TITLE 1 (1)	--	Title of station A
TITLE 1 (2)	--	Title of station B

Constants

CON	P/N	Period of revolution/number of occultations per revolution
DTRAD	$\pi/180$	Conversion of degrees to radians
F2	f^2	Focal length of camera squared
PI	π	3.141592654

Trail Equation

EN,N	n	Number of points used to get trail equation
SLOPE	m	Slope of line found by trail equation
SUMX	ΣX	Sum of X readings
SUMX2	ΣX^2	Sum of X readings squared
SUMXY	ΣXY	Sum of the product of the X and Y readings
SUMY	ΣY	Sum of the Y readings
XØ	X_o	First X reading on trail
YØ	Y_o	Average Y reading on trail

XTR	ΔX	Difference between actual reading of point and X_o, Y_o for points on trail
YTR	ΔY	
XBAR	$\overline{X}$	Average value of X reading
YBAR	$\overline{Y}$	Average value of Y reading
ZTRAIL	--	Y intercept of trail equation

Relative Coordinates

ELEV	--	Elevation of station above mean sea level
ELONH	--	Geographic longitude of station in hours, minutes, and seconds respectively
ELONM	--	
ELONS	--	
ELON	L	Geographic longitude of station in hours
ELZ,EMZ,ENZ	λ_z,μ_z,ν_z	Direction cosines of zenith of station
EXIABO, ETAABO, ZETAAB	$\xi_{AB_o},\eta_{AB_o},\zeta_{AB_o}$	Direction cosines of vector from station A to station B at sidereal time zero at station A and at the time of the event, respectively
EXIAB, ETAAB, ZETAAB	$\xi_{AB},\eta_{AB},\zeta_{AB}$	Direction cosines of vector from station A to station B at time of event
GTHR,GTM,GTS	--	Given event Greenwich time in hours, minutes, and seconds
PHD,PHM,PHS	--	Geographic latitude of station in degrees, minutes, and seconds
PHI	ϕ	Geographic latitude of station in radians

CPHI,SPHI	$\cos \phi$, $\sin \phi$	Cosine and sine of latitude
PHIP,PHIPD	ϕ'	Geocentric latitude of station in radians and degrees respectively
RAB	R_{AB}	Range from station A to station B
RHO	ρ	Earth's radius at the station
THETØH,THETØM,THETØS	θ_o	Sidereal time at 0000 U.T. at Greenwich for the date of the event in hours, minutes, and seconds
DTHETS	$\Delta\theta$	Correction in sidereal time (seconds)
THET,THETA	θ	Sidereal time in degrees or radians
CTH,STH	$\cos \theta$, $\sin \theta$	Cosine and sine of theta
UT	--	Universal time of event
DL	ΔL	Difference in longitudes of the two stations
CDL	$\cos \Delta L$	Cosine and sine of ΔL
SDL	$\sin \Delta L$	

Calculation of Pole and Radiant

CAX,CBX,CCX	a_x, b_x, c_x	Inverse plate constants for six constant solution (or four constant solution expanded to give six numbers)
CAY,CBY,CCY	a_y, b_y, c_y	
EXIBAR,ETABAR	$\bar{\xi}, \bar{\eta}$	Standard plate coordinates of a point
X	X	Measured reading of X on plate
Y	Y	Computed Y of point read at X

ELC,EMC,ENC	λ_c, μ_c, ν_c	Direction cosines of plate center declination-right ascension system
ELETA,EMETA,ENETA	$\lambda_\eta, \mu_\eta, \nu_\eta$	Direction cosines of $\bar{\eta}$ axis declination-right ascension system
ELEXI,EMEXI	λ_ξ, μ_ξ	Direction cosines of $\bar{\xi}$ axis declination-right ascension system
EL,EM,EN	λ, μ, ν	Direction cosines of any point on trail declination-right ascension system
ELP,EMP,ENP	λ_p, μ_p, ν_p	Direction cosines of pole declination-right ascension system
DLONG	l	Trail length in degrees
SINL	$\sin l$	Sine of trail length
SINQ	$\sin Q$	Sine of angle of intersection between the two trails as seen in the sky from stations A and B
ELR,EMR,ENR	λ_R, μ_R, ν_R	Direction cosines of the radiant

Calculation of Ranges, Heights, and Distances

XP	X_i	X reading of any point
YP	Y_i	Corresponding Y computed for this X point
COSZ,COSZ1	$\cos Z$	Cosine of zenith distance of any point
D	D	Distance along the trail from an arbitrary zero
PEL,PEM,PEN	λ, μ, ν	Direction cosines of each point in declination-right ascension system

HEIGHT, H	H	Height above mean sea level for the point
DH	δH	Height correction
H1	h	Height above the tangent plane
RANGE, R	R	Range from the station to the point
NUM	--	Number of dashes
NDASH,ENDASH	--	Number of the dash (numbering from 1-NUM)
NWT	--	Weight or reliability of dash
EXIP,ETAP,ZETAP	--	Direction components of point declination-right ascension system

Calculation of relative time

XQ,YQ	X_q, Y_q	Observed coordinates of the center of rotation of the shutter
XC,YC	X_c, Y_c	Observed coordinates of the projection center
DX,DY	--	Differences in two centers
XMEAS	X	X reading of any point
DYTR	--	Difference between Y computed and Y of rotation center
OMEGA	ω	Angle that blade makes with dash
OCCULT	N	Number of occultations per revolution
P	P	Period of revolution

SIGN, IF PLUS	--	Code describing direction of revolution of shutter
IFEXT	--	Code if cards are desired
ELAZ,EMAZ,ENAZ	ξ,η,ζ	Direction cosines of point in the altitude-azimuth system

I. INTRODUCTION

The optical reduction programs for the 7094 computer have been written to compute the space trajectory of the re-entering bodies of the Trail-blazer experiments fired from Wallops Island, Virginia. When a body re-enters the atmosphere it is heated and may become luminous; thus, it may be photographed. Trails are recorded photographically from several stations by Lincoln Laboratory and the NASA and the photographs are available for analysis. The programs discussed in this report compute positions along the re-entry trail using known stars as calibrations. Relative time may be introduced by the use of chopping shutters on one or more cameras, so that breaks at regular time intervals are produced along the trail.

The mathematical background of the reduction procedure is discussed in Section II and the experimental techniques in Section III. The programs themselves are documented in Sections IV and V. The astrometric reduction is modeled after the technique discussed by Whipple and Jacchia[1] and McCrosky[2]. The programs are adapted from programs written by Dr. McCrosky's group at the Harvard Observatory for meteor analysis.

A camera usually maps the celestial sphere onto a flat focal plane. In the case of the Super-Schmidt Cameras, the focal plane is curved and the film must be copied by projection onto a flat plate for analysis. All measurements for this reduction are made on the flat plates. Positions on the plate are measured as discussed in Section III.

The angular direction to the reference stars and the vehicle trail are determined with respect to each camera station, and from these directions the angular bearing of measured points along the trail can be found. The trails from the two stations may look quite different since they are photographed at different distances and angles. Also, clouds sometimes obscure parts of the trail. Thus the beginning and end points of the trails recorded from the two stations may not be identical. No attempt has been made to synchronize the chopping shutters in the Wallops Island camera systems.

Since the Trailblazer vehicles are axially symmetric, the only force acting to change the direction of the re-entry from a straight line is the gravity effect on a freely falling body. This effect is generally small over the short time of the event and the deviation from a straight line trajectory, if any, would be found near the end of the trail. Also the re-entry body of the Trailblazer system is fired downward and re-enters about 10-12° from the vertical so that gravity acts mainly along the vehicle trajectory. Thus the re-entry trail is usually a straight line on the plate. If the trail is curved, it must be broken up into straight line segments and the program must be run separately for each straight line part.

From the known positions of stars near the trail at the time of the event, a calibration is performed on each plate; from the results of this calibration, the angular position of points along the trail may be determined for each plate. Since the trail appears as a straight line, the position vectors of points along the trail as seen from the camera station must determine a unique plane. The re-entry must lie on the intersection of the two planes determined from a pair of photographs taken at separate stations. The distance from the camera site to the point on the line of intersection may be computed individually for each point. Therefore, a set of slant ranges R_i for points along the trail may be found for each plate. Height, direction cosines, and relative distance along the trail may then be computed for each measured point along the trail. If the plates were obtained from a camera with a chopping shutter, relative time may be computed for each dash.

In general a unique time for any given point on the plate can not be found. Also, no exact correlation exists between points on the two trails. If a common point such as a flare can be identified on both plates, the points may be considered to occur at the same time. The height of the point computed from both stations should be identical (this will serve as a check on the height computations). In this case, also, the exact sidereal time of the flare can be computed; however, since in general flares need not occur, this computation of sidereal time is not included in the present program. In the Project PRESS coded shutters have been built into the cameras and the chopping shutters have been synchronized so that absolute time can be recorded and an analysis performed using unique time points from two or more stations. The Super-Schmidt Cameras used in the Trailblazer program are not equipped to record absolute time since suitable coded shutters can not be easily installed. It should be noted that the re-entry events of Project PRESS are much longer than those of the Trailblazer series; thus the design and use of absolute timing equipment presents much less difficulty in Project PRESS.

The present set of programs is designed to compute the position of the body, i.e. height, range, and direction cosines of each point along the trail. If the trail is chopped, relative times can be computed for the dashes; relative distances along the trail from an arbitrary point on the trail are also computed. These programs do not compute velocity. If the distance along the trail as a function of time is known, various methods of computing velocity might be used. Harvard has obtained good results in meteor work with the equation

$$D = a + bt + c \exp (kt)$$

where D is the distance along the trail; t is the elapsed time from an arbitrary reference point on the trail and a, b, c and k are computed constants. However, this fit did not appear particularly suitable for the Trailblazer work. At present velocity is computed from the results of the optical trajectory program by hand computation. Since this method is not yet suitable for machine calculation, the computation of velocity has been omitted from the present program.

II. MATHEMATICAL BACKGROUND

A. Astronomical Coordinate Systems

When the stars are observed from a point on earth they appear to lie on the surface of a sphere with the observer at its center. Although the eye can not determine the distance to the stars, an estimate of the angles subtended at the observer by any pair of stars can easily be made. These directions are defined in terms of the positions on the surface of a sphere - the celestial sphere - in which the straight lines joining the observer to the stars intersect this surface. The axis of the earth pierces the sphere at two points called "the celestial poles". The radius of the celestial sphere is considered arbitrarily large.

The position of any point on the surface of a sphere may be completely specified by reference to two principal great circles on the sphere, or by one great circle and a point on that great circle. Thus a point on the surface of the earth is completely specified by its latitude, measured from the equator, and its longitude, measured east or west from Greenwich. Several methods of defining position on the celestial sphere are in common use.

1. Spherical Coordinate Systems

a. The Altitude-Azimuth System

Let O, the observer on the surface of the earth, be the center of the celestial sphere. Z, the zenith, is the point directly overhead (defined by a plumb line) and the nadir is the point directly underfoot. The plane through O perpendicular to the direction of the zenith is called the plane of the horizon and, as shown in Fig. 1a, cuts the celestial sphere in the great circle NAS dividing the celestial sphere into two hemispheres, the upper being visible and the lower being hidden by the earth itself. A vertical circle is a great circle passing through the zenith and the nadir.

Let X be the position of a star on the celestial sphere at a given moment. The angle $\widehat{AOX}$ or the great circle arc AX is called the altitude, a, of the star. To completely specify the position of a point on the celestial sphere the particular vertical circle on which it lies must be specified. Let OP be parallel to the earth's axis. If the observer is in a northern latitude the position P is called the north celestial pole. The vertical circle through the pole, ZPN in Fig. 1a, is defined as the principal vertical; it is the circle traced on the celestial sphere by the observer's meridian. The point where this circle cuts the horizon is called the north point, N, of the horizon plane; the point S directly opposite N on the horizon is the south point.

If the star is in the eastern* part of the sphere, as is X in Fig. 1a, the spherical angle $\widehat{PZX}$ or the great circle arc NA is called the azimuth A; if the star is in the western part of the sky, as is Y, the azimuth is the angle $\widehat{PZY}$ which in this case is greater than 180°. Throughout this report azimuth will always be measured from north in an easterly direction from 0-360°. Since the angle $\widehat{POZ}$ in Fig. 1a equals the angle between the radius of the earth which passes through the observer's position and the earth's axis, $\widehat{POZ}$ is seen to be equal to the co-latitude of the observer.

b. The Declination-Hour Angle System

The celestial equator is the great circle of the celestial sphere perpendicular to the polar axis and is thus the great circle in which the plane of the earth's equator cuts the celestial sphere, RWT in Fig. 1b. The celestial equator and the horizon intersect at two points E and W which are each 90° from N and S and are called the east and west points.

Let X be a star in the northern hemisphere. If PXD is the semi-great circle through X and the pole P, the arc DX is defined as the declination δ of X, i.e. its distance in degrees north (+) or south (-) of the celestial equator. A great circle passing through the pole of the celestial sphere

*If the observer faces north, the west is on his left and the east on his right.

is called a meridian or an hour circle. The second great circle required for complete specification of the star's position in the declination-hour angle system is taken as the observer's meridian (i.e. that meridian passing through the zenith) PZRSQ (L) in Fig. 1b. At any moment the position of X is specified by the angle at the pole between the observer's meridian and the meridian PXQ through the star at that time. This angle $R\hat{P}X$ or $Z\hat{P}X$ or the arc RD is called the hour angle H and is measured from the observer's meridian westward 0° -360° or 0h -24h.

c. The Declination-Right Ascension System

In the hour angle-declination system only one coordinate, the declination, remains constant as the earth rotates whereas the hour angle increases uniformly from 0h -24h during a day. It is usually more convenient to define the second coordinate relative to a fixed point on the equator (although all the stars appear to change position during the daily rotation of the earth, their relative positions with respect to one another remain unchanged over long periods of time). The point chosen as standard is the vernal equinox or the first point of Aries denoted by Υ. Then the arc ΥD or angle $\Upsilon\hat{P}X$ is called the right ascension of star X, denoted by α, and is measured eastward from 0h -24h (or from 0 -360°) Fig. 1b. Note: right ascension is measured in the opposite direction to hour angle.

d. The Celestial Latitude - Longitude System

The earth is a planet rotating around the sun in an elliptical orbit with the sun at one focus with a period of revolution of approximately one year. During the year, as observed from earth, the sun appears to make a complete circuit of the sky against the star background. The plane of this orbit is called the plane of the ecliptic and the great circle in which this plane cuts the celestial sphere is called the ecliptic. This plane is found to be inclined at an angle of about 23 1/2° to the celestial equator; this inclination is called the obliquity of the ecliptic, denoted by ϵ; $\epsilon = M\hat{\tau}R$ in Fig. 1c.

Relative to the earth the sun appears to move on the surface of the celestial sphere along the ecliptic in the direction $Y\tau M$, and twice a year its position on the celestial sphere coincides with the intersection of the ecliptic with the celestial equator. This position τ at which the sun's declination changes from south to north is defined as the vernal equinox; it is the reference point in the right ascension-declination system.

A fourth set of coordinates can be derived using the ecliptic as a fundamental great circle and the vernal equinox τ as a principal reference point. In Fig. 1c let K be the pole of the ecliptic and KXA a great circle arc passing through star X meeting the ecliptic in A. The arc τA measured from τ to A along the ecliptic in the direction of the sun's annual motion, i.e. eastward, is called its celestial longitude λ and is measured from 0 -360°. The arc AX is called the celestial latitude β and is measured north or south of the ecliptic.

e. Notes on Coordinate Systems

In the previous definitions, the center of the celestial sphere was taken as the observer on the surface of the earth. However, since the stars are at distances which are almost infinitely large as compared with the dimensions of the earth, no appreciable error results from taking the center of the earth as the center of the celestial sphere; this convention will be used in this report when dealing with the star background. Although our figures have been drawn for the northern hemisphere, similar figures apply for the southern and the definitions of the coordinate systems apply to both hemispheres. For an observer exactly at the pole the definitions of azimuth and meridian break down; thus in the following study it will be assumed that the observer is not at a pole.

2. Rectangular Coordinate Systems

In the preceding discussion, points on the celestial sphere were defined in terms of spherical coordinates; however, in most of the following work it is much more convenient to use a rectangular reference system. A right handed coordinate system is chosen in which the coordinates x, y, z are measured along three perpendicular axes such that the rotation of the x axis into the y axis around the z axis is accomplished by the right hand rule; the origin of the system is taken as the observer or the earth's center (see above). The position of a star on the celestial sphere may be represented in vector form by

$$\bar{r} = r_x \bar{i} + r_y \bar{j} + r_z \bar{k} \tag{1}$$

where $\bar{i}$, $\bar{j}$, and $\bar{k}$ are unit vectors in the x, y, z directions respectively, and r_x, r_y and r_z are the projections of $\bar{r}$ on the respective axes. The cosines of the angles between the positive x, y, and z axes and the vector $\bar{r}$ measured in the direction from the axis toward $\bar{r}$ are called the direction cosines of $\bar{r}$. We will express all coordinates in terms of direction cosines; in this way the magnitude of $\bar{r}$ will not appear and only the directions will be used.

a. The Altitude-Azimuth System

The horizon will be taken as the xy plane, the positive x axis will pass through the east point of the horizon, the positive y axis through the north point, and the positive z axis through the zenith.

Let S in Fig. 2a be any star on the celestial sphere with altitude a and azimuth A. Then using ξ, η, ζ to represent direction cosines in the x, y, z directions, respectively

$$\begin{aligned} \xi &= \cos a \sin A \\ \eta &= \cos a \cos A \\ \zeta &= \sin a \end{aligned} \tag{2}$$

b. The Declination-Hour Angle System

The plane of the equator will be taken as the xy plane, the positive x axis will pass through the west point on the equator, the positive y axis through the intersection of the observer's meridian and the equator, and the positive z axis through the celestial north pole. Thus, in the form of direction cosines ℓ, m, n (Fig. 2b)

$$\ell = \cos\delta \sin H$$
$$m = \cos\delta \cos H$$
$$n = \sin\delta. \qquad (3)$$

c. The Declination-Right Ascension System

The plane of the equator will be taken as the xy plane; the positive x axis will pass through the vernal equinox τ, the positive y axis through the point whose right ascension is +90°, and the positive z axis through the celestial north pole. Thus (Fig. 2c)

$$\lambda = \cos\delta \cos\alpha$$
$$\mu = \cos\delta \sin\alpha$$
$$\nu = \sin\delta \qquad (4)$$

where λ, μ, ν are direction cosines in the x, y, z directions respectively.

B. Precession and Proper Motion

Although the phenomenon of precession was observed by Hipparchus over 2000 years ago, the first correct dynamical explanation was given by Newton. As is well known the earth has the shape of an oblate spheroid rather than that of a sphere due to an equatorial bulge produced by its rotation about its polar axis; this bulge is inclined to the plane of the ecliptic by about 23 1/2°. Thus the solar and lunar attractions produce moments about the center. Since the earth is spinning rapidly on its axis much like a top, instead of turning the equatorial plane into coincidence with the ecliptic plane, the result of these attractive forces is to cause the axis of the earth to precess about the axis of the ecliptic. Thus the earth's axis has a conical motion about the pole of the ecliptic so that the pole of the equator describes a small circle about the ecliptic pole and the vernal equinox moves backwards along the ecliptic.

The precession of the equinoxes causes the celestial longitude of a star to increase at the rate of about 50 seconds of arc per year while the celestial latitude shows no significant change. In our current study we are interested primarily in the effect of precession on the right ascension and declination of a star. When the coordinates of the star are expressed in direction cosine form precession may be considered the result of a rotation of the axes of the system with the origin unchanged. Let us consider the effect on a star S with direction cosines λ, μ, ν relative to the original axes X, Y, Z of a rotation of these axes to a new position X', Y', Z' where the angles between OX', OY', OZ' and OX, OY, OZ are shown in Table I where, for example, X_z represents the angle between OX' and OZ.

TABLE I

	OX	OY	OZ
OX'	X_x	X_y	X_z
OY'	Y_x	Y_y	Y_z
OZ'	Z_x	Z_y	Z_z

Thus the transformation is given by the matrix equation

$$\begin{pmatrix} \lambda' \\ \mu' \\ \nu' \end{pmatrix} = \begin{pmatrix} X_x & X_y & X_z \\ Y_x & Y_y & Y_z \\ Z_x & Z_y & Z_z \end{pmatrix} \begin{pmatrix} \lambda \\ \mu \\ \nu \end{pmatrix} \qquad (5)$$

The direction cosines λ', μ', ν' giving the position of a star at any given time can be easily obtained from the direction cosines λ, μ, ν at a standard equinox if the values of the transformation matrix are known. Usually the right ascension and declination of a star are tabulated for the equinox of 1950 in star catalogues; the values of the transformation matrix can be found[3].

In 1718 Halley discovered that the position of certain bright stars had changed appreciably in relation to the general star background since the time of Hipparchus. This suggested that these stars had a definite space velocity relative to the sun and lay at finite distances from the earth. The proper motion of a star is defined as its apparent angular rate of motion on the celestial sphere; it is usually measured in seconds of arc per year. The proper motion is usually found by comparing precise observations of the right ascension and declination of the star taken many years apart. The star catalogues give the components μ_α, the proper motion in right ascension, and μ_δ, the proper motion in declination, per year or per century for each star tabulated. Since the components of proper motion depend upon the epoch and equinox for which they are measured, it is important to apply proper motion corrections to the tabulated values of α and δ for a star before precession is applied. Thus, for example, suppose the right ascension and declination of star S in 1935 referred to the equinox of 1950.0 is known and it is desired to compute the coordinates of the star in 1964 referred to the 1964.0 equinox. The simplest method is to first apply proper motion corrections from the tabulated values to find the position of S in 1964 relative to the equinox of 1950.0 and then to precess the star to the eqinox of 1964. A detailed discussion of precession and proper motion may be found in Smart[4].

Another effect of some interest is nutation which is a slight periodic oscillation of the actual pole around the mean pole where the mean pole is precessing uniformly. Since the effects of this nutation are small no corrections for them appear in this work.

C. Time

The sidereal time at any place on earth at any instant is, by definition, the hour angle of the vernal equinox at that instant. When Υ is on the observer's meridian the sidereal time is 0; when τ is next on the observer's meridian after 24h of sidereal time or one complete revolution of the earth about its axis, a sidereal day has passed. From Fig. 1b it is obvious that

$$R\tau = RD + \tau D$$

Now RD is the hour angle of star X and α is its right ascension; moreover $R\tau$ is the hour angle of τ. Therefore

$$\text{Sidereal time (observer)} = \text{hour angle X} + \text{right ascension X}$$

$$S.T. = H + \alpha \qquad (6)$$

Now let G on the celestial sphere in Fig. 1b be the zenith of Greenwich; $\widehat{GPZ}$ is the longitude of the observer, $\widehat{GPX}$ the hour angle of X from Greenwich and $\widehat{ZPX}$ the hour angle of X from the observer's meridian $\angle$. Since

$$\widehat{GPX} = \widehat{ZPX} - \widehat{ZPG} = \widehat{ZPX} - \text{longitude of observer}$$

Obviously:

$$\text{H.A. of X at Greenwich} = \text{H.A. of X at } \angle \pm \text{longitude of } \angle$$

Since this relation is general it also holds for τ. Thus

$$\text{S.T. at Greenwich} = \text{S.T. at } \angle \pm \text{longitude at } \angle \qquad (7)$$

The sign of $\angle$ is + when $\angle$ is west of Greenwich and - when $\angle$ is east of Greenwich.

A mean solar day is defined as the interval between two successive transits of a mean sun* across the observer's meridian and is the day used in our system of civil time. A sidereal day has been found to be 3m 56.556s shorter than a mean solar day.

---

*The mean sun is a fictitious body which is defined to move in the celestial equator at a uniform rate around the earth so that it completes a revolution in the same time as the true sun completes a circuit of the ecliptic.

In our later work it will often be necessary to compute the sidereal time at a point on earth at a given local time. It is most convenient to compute the sidereal time at Greenwich first and then to use equation 7 to find local sidereal time. The American Ephemeris and Nautical Almanac[5] for the proper year contains a table called "Universal and Sidereal Times" which gives the sidereal time for each day of the year for 0h Greenwich time for that day, i.e., the sidereal time at midnight at Greenwich for the given day; each day starts at a different sidereal time since the length of a day in the two systems is different. The given local time is changed into Universal time by adding the proper correction (+5 hours for E.S.T.). The sidereal time at Greenwich is next calculated by

$$\theta_G = \theta_o + \text{U.T.} + \Delta\theta \tag{8}$$

where θ_G is the sidereal time at Greenwich, θ_o is the sidereal time at 0h UT taken from Ref. 5 and $\Delta\theta$ is the correction required since a solar day is longer than a sidereal day.

$$\Delta\theta = 3\text{m } 56.556\text{s } (\text{U.T.h}/24\text{h}) = 236.556 \ (\text{U.T.h}/24\text{h}) \text{ sec} \tag{9}$$

To find the sidereal time at station A from its value at Greenwich at the same time we merely use equation 7 and the whole equation then becomes

$$\theta_A = \theta_o + \text{UT} - L_A + \Delta\theta \tag{10}$$

where θ_A is the desired time at A and L_A is the longitude of A (west in this case).

D. Transformation of Coordinates

In our later work it will be necessary to transform between the altitude - azimuth and the declination - right ascension systems. Let us consider the problem of converting from the right ascension-declination system to the altitude-azimuth system in direction cosine form. We are given α and δ for star X at a given time t and a given latitude ϕ. We must first transform our three direction cosines λ, μ, ν, to the l, m, n cosines of the declination-hour angle system (Fig. 3a).

It is obvious that $n = \nu$ since the axis is identical for the two systems and the x, y plane is the same in both cases.

$\theta = \alpha + H$ where θ is sidereal time at the observer so that the transformation may be seen to take place as in Fig. 3b where the rotation of axes in the x,y plane is shown.

$$
\begin{aligned}
l &= \lambda \cos(90 - \theta) - \mu \sin(90 - \theta) = \lambda \sin\theta - \mu \cos\theta \\
m &= \lambda \sin(90 - \theta) + \mu \cos(90 - \theta) = \lambda \cos\theta + \mu \sin\theta \qquad (11) \\
n &= \nu
\end{aligned}
$$

Next, we must transform from the declination-hour angle system to the altitude-azimuth system (Fig. 3c). The y,z -plane is common to both systems and $x_3 = - x_2$, so that immediately

$$\xi = \mu \cos\theta - \lambda \sin\theta$$

Also, $PZ = 90 - \phi$ so that the transformation may be represented in the plane as in Fig. 3d.

$$
\begin{aligned}
\xi &= \mu \cos\theta - \lambda \sin\theta \\
\eta &= -\left[m \cos(90 - \phi) - n \sin(90 - \phi) \right] = \nu \cos\phi - (\lambda \cos\theta + \mu \sin\theta) \sin\phi \\
\zeta &= m \sin(90 - \phi) + n \cos(90 - \phi) = (\lambda \cos\theta + \mu \sin\theta) \cos\phi + \nu \sin\phi
\end{aligned}
\qquad (12)
$$

E. Astronomical Photography

A section of the celestial sphere can be photographed by using the general configuration of Fig. 4. Let CO be the optical axis of a lens with focal length f and assume that CO passes through the geometric center of the plate. The focal plane of the lens, FG, is perpendicular to OC. Let us consider a star on the optical axis at C; all the rays from this star falling on the lens will be brought to a focus at O and an image formed at that point. If another star lies on the celestial sphere at S, it is seen to form an image at R. If the astronomical coordinates of C and S are known, since f is the known focal length of the lens and $\sigma = b$, we can find the relation between the linear distance d on the photographic plate and the corresponding angular distance σ in the sky. Thus

$$d = f \tan b = f \tan \sigma \tag{13}$$

We will now define a system of axes on the plate through the plate center such that the position of a star on the plate can be related to its astronomical coordinates. First, the direction cosines for the plate center in the right ascension-declination system are found by applying equation 4 to the special case of the plate center.

$$\begin{aligned} \lambda_c &= \cos \delta_c \cos \alpha_c \\ \mu_c &= \cos \delta_c \sin \alpha_c \\ \nu_c &= \sin \delta_c \end{aligned} \tag{14}$$

Standard coordinates $\bar{\xi}$ and $\bar{\eta}$ will be rectangular coordinates in the plane tangent to the celestial sphere at C with their origin at C such that $\bar{\eta}$ increases northerly along the hour circle through C and $\bar{\xi}$ increases toward the east tangent to the parallel of declination of C (Fig. 5).

Having found the direction cosines of the plate center in terms of the declination right-ascension system, it is now desired to determine the direction cosines of the plate axes $\bar{\xi}$ and $\bar{\eta}$, defined above, using the same system. The angle OCP' is a right angle since $\bar{\eta}$ on the sphere must be perpendicular to a radius OC of the sphere. Therefore, OP'C must equal δ_c and so $\nu_\eta = \cos \delta_c$. Also the projection of $\bar{\eta}$ on the equatorial plane must be $-\sin \delta_c$ so that the direction cosines of $\bar{\eta}$ are given by

$$\begin{aligned} \lambda_\eta &= -\sin \delta_c \cos \alpha_c \\ \mu_\eta &= -\sin \delta_c \sin \alpha_c \\ \nu_\eta &= \cos \delta_c \end{aligned} \tag{15}$$

Since $\bar{\xi}$ increases along a tangent to a parallel of declination, its projection on the z axis is zero so that $\nu_\xi = 0$. The $\bar{\xi}$ axis is directed eastward from the point on the sphere representing the plate center; hence its direction is $\alpha_c + 90°$ (Fig. 5). Hence:

$$\lambda_{\xi} = -\sin \alpha_c$$
$$\mu_{\xi} = +\cos \alpha_c$$
$$\nu_{\xi} = 0 \tag{16}$$

Now let S be a star on the celestial sphere with coordinates (α_s, δ_s) and direction cosines in our normal equatorial system $(\lambda_s, \mu_s, \nu_s)$. Let us find the direction cosines of S relative to the coordinate system of the photographic plate $(\bar{\xi}, \bar{\eta})$. The direction cosines of the standard plate system's axes relative to the equatorial axes are shown in Fig. 6b. Thus the direction cosines of S in the plate-centered system are

$$P_{\bar{\xi}} = \lambda_{\xi}\lambda_s + \mu_{\xi}\mu_s$$
$$P_{\bar{\eta}} = \lambda_{\eta}\lambda_s + \mu_{\eta}\mu_s + \nu_{\eta}\nu_s$$
$$P_z = \lambda_c\lambda_s + \mu_c\mu_s + \nu_c\nu_s \tag{17}$$

It is also true that

$$\cos \sigma_s = \lambda_c\lambda_s + \mu_c\mu_s + \nu_c\nu_s \tag{18}$$

where σ_s is the angle between the star and the plate center. From Fig. 4 it can be seen that the star located on the celestial sphere at S is seen by the camera at R, as if it were located on the tangent plane at D. This type of projection is called a gnomonic projection and it is assumed that the camera lens is located at the center of the celestial sphere. The gnomonic projection is accomplished by dividing equation 17 by the cosine of σ_s. One then obtains the pair of standard plate equations for the star S:

$$\bar{\xi}_s = (\lambda_{\xi}\lambda_s + \mu_{\xi}\mu_s) / \cos\sigma_s$$
$$\bar{\eta}_s = (\lambda_{\eta}\lambda_s + \mu_{\eta}\mu_s + \nu_{\eta}\nu_s) / \cos\sigma_s \tag{19}$$

F. Plate Constants

Let us now assume that we have a photographic plate containing a re-entry trail against the star background with the camera driven by an equatorial mount so that the stars appear as points on the plate. The trail of the re-entry object will appear as a broken line if a chopping shutter is used.

The first part of the analysis will be the calibration of the plate. This will allow us to find the right ascension and declination of any point on the plate (at least in the region of interest) when the X and Y measurements on the plate are known. A number of stars (probably about 6-15) are identified near the trail; their right ascensions and declinations are found from standard tables for a given date. After corrections are made for proper motion and precession on the tabulated coordinates, we have exact values of α_s and δ_s for each star at the time of the event *, so that we now know the astronomical coordinates of points on our plate. An approximate position, α_c and δ_c for the plate center is required so that the standard plate coordinates $\bar{\xi}_s$ and $\bar{\eta}_s$ can be calculated for each star using equation 19.

Two methods of relating α_s, δ_s of the star and the X,Y readings on the plate are in common use. The measurements are taken so that the X-axis is parallel to the trail of the re-entry and the Y-axis is perpendicular to the X. In the "four constant solution" we solve the two equations

$$\begin{aligned} a\,\bar{\xi}_s + b\,\bar{\eta}_s + c &= X_s \\ b\,\bar{\xi}_s - a\,\bar{\eta}_s + d &= Y_s \end{aligned} \qquad (20)$$

Two stars at the ends of the trail are used to compute the four constants a,b,c and d and the standard coordinates of any point on the trail may then be found from the inverse transforms:

$$\begin{aligned} \bar{\xi}_s &= A\,X_s + B\,Y_s + C \\ \bar{\eta}_s &= B\,X_s - A\,Y_s + D \end{aligned} \qquad (21)$$

where:

$$A = a / (a^2 + b^2) \qquad B = b / (a^2 + b^2)$$

$$C = -(ac + bd) / (a^2 + b^2) \qquad D = (ad - bc) / (a^2 + b^2)$$

*Proper motion is corrected to the nearest month and precession to the nearest 1 January.

The "six constant solution" requires a solution of the equations

$$a_x \bar{\xi}_s + b_x \bar{\eta}_s + c_x = X_s; \; a_y \bar{\xi}_s + b_y \bar{\eta}_s + c_y = Y_s \qquad (22)$$

for the constants a_x, b_x, c_x, a_y, b_y, and c_y by a least squares method. The standard coordinates can then be found by

$$\bar{\xi} = a_\xi X + b_\xi Y + c_\xi$$

$$\bar{\eta} = a_\eta X + b_\eta Y + c_\eta$$

where:

$$a_\xi = b_y/c_{xy} \qquad a_\eta = -a_y/c_{xy}$$

$$b_\xi = -b_x/c_{xy} \qquad b_\eta = a_x/c_{xy}$$

$$c_\xi = (-b_y c_x + b_x c_y)/c_{xy} \qquad c_\eta = (a_y c_x - a_x c_y)/c_{xy}$$

$$c_{xy} = a_x b_y - a_y b_x \qquad (23)$$

The six constant solution is more complex. However it has the advantage that the X and Y measuring axes need not be exactly perpendicular so that it can be used to correct for any non-orthogonality of a measuring engine. It will also correct for any linear distortions on the plate.

When the plate constants have been found a process of second order corrections is usually applied. These corrections make allowances for errors caused by optical distortions, distortions caused by the emulsion, misplacement of the plate center, errors in the screws, etc. It is now possible to compute theoretical values of X and Y for each star -- i.e., the position where a star of the given right ascension and declination should lie on the plate. From these values residuals are computed from the equations:

$$\Delta X_s = X_s\,(\text{comp}) - X_s\,(\text{obs}) \qquad \Delta Y_s = Y_s\,(\text{comp}) - Y_s\,(\text{obs}) \qquad (23')$$

The observed values of X and Y for points on the trail are usually corrected by the equations:

$$X\,(\text{corr}) = X\,(\text{meas}) + \Delta X\,(\text{curve}) \qquad Y\,(\text{corr}) = Y\,(\text{meas}) + \Delta Y\,(\text{curve})$$

where ΔX (curve) and ΔY (curve) are values found from a smooth curve through the residuals ΔX and ΔY. The measured X is the abscissa for the correction curves and the ΔX or ΔY is the ordinate. By examining the residuals it is usually easy to spot an obviously misidentified or misread star since its residuals will be much larger than normal. Often one or more stars can be eliminated and the calibration rerun.

When the plate calibration has been completed, it is possible to relate the X, Y readings of any point on the plate to a position on the celestial sphere. First the standard plate coordinates are found by equation 21 or 23. Direction cosines of the point can then be obtained directly by:

$$\lambda = (\lambda_c + \lambda_\eta \bar{\eta} + \lambda_\xi \bar{\xi}) / (1 + \bar{\xi}^2 + \bar{\eta}^2)^{1/2}$$

$$\mu = (\mu_c + \mu_\eta \bar{\eta} + \mu_\xi \bar{\xi}) / (1 + \bar{\xi}^2 + \bar{\eta}^2)^{1/2}$$

$$\nu = (\nu_c + \nu_\eta \bar{\eta}) / (1 + \bar{\xi}^2 + \bar{\eta}^2)^{1/2} \qquad (24)$$

G. The Trail Equation

It is assumed in the following reduction that the re-entering vehicle travels in such a path that its projection on the plate is a straight line. If this is not the case the reduction must be done separately on straight line segments of the trail. It is therefore possible to compute a straight line of the form

$$Y = mX + b \qquad (25)$$

where m is the slope of the line and b the y intercept; thus only X need be measured directly from the plate after the equation of the line is found. Reading to compute the trail equation is done by dividing the trail into a suitable number of equally spaced (usually between 10 and 20) sections in X and reading the Y corresponding to these readings. Then the X and Y measurements are related by a least squares formula

$$m = (\Sigma X_i Y_i - n \bar{X} \bar{Y}) / (\Sigma X_i^2 - n \bar{X}^2) \qquad b = \bar{Y} - m \bar{X} \qquad (26)$$

where n is the number of points read and $\bar{Y}$ and $\bar{X}$ are average values.

H. The Pole of the Trail

As discussed above the vehicle appears to travel in a great circle if the trail is straight along the plate. If the trail is not straight the pole is computed by using points from the early straight line part. The pole of a great circle is, by definition, a point 90° from each point on the great circle. In computing the pole of a trail we will use the beginning and end points (a middle point may be used as a check). It is readily seen that the pole of a great circle through the trail can be found from the cross product of the beginning and end points on the trail. Hence, the direction cosines of the pole $(\lambda_p, \mu_p, \nu_p)$ are given by

$$\lambda_p \sin \ell = \mu_b \nu_e - \nu_b \mu_e$$

$$\mu_p \sin \ell = \nu_b \lambda_e - \lambda_b \nu_e$$

$$\nu_p \sin \ell = \lambda_b \mu_e - \mu_b \lambda_e \qquad (27)$$

where λ_b, μ_b, ν_b are direction cosines of the beginning point of the trail and λ_e, μ_e, ν_e those of the end point, and ℓ is the length of the trail (in angular measure).

I. Relative Coordinates of the Two Stations

It is necessary to use two photographic plates taken from two separate stations to perform a complete optical reduction. These stations will hereafter be called station A and station B. We will now compute the relative positions of the two stations with respect to each other. We will assume that we know their geographic latitudes and longitudes, their elevations above sea level and the exact time of the event. The coordinate system will be that shown in Fig. 7. We must first find the geocentric latitude* ϕ' of the two stations since we are working in a geocentric system; we will also find the earth's radii R_A and R_B for stations A and B, respectively, using the international ellipsoid. We find[5]

$$\phi' = \phi - 11'35."6635 \sin 2\phi + 1."1731 \sin 4\phi - 0."0026 \sin 6\phi$$

*Geographic latitude is measured at the earth's surface with reference to the local vertical; geocentric latitude is measured from the center of the earth.

$$\rho = a(0.998320047 + 0.001683494 \cos 2\phi - 0.000003549 \cos 4\phi + 0.000000008 \cos 6\phi)$$
$$a = 6378.388 \text{ Km} = 20926.42796 \text{ Kft}$$
$$R = \rho + \text{elevation of station} \qquad (28 \text{ contd})$$

Letting ϕ_A, ϕ_B be the geographic latitudes, R_A,R_B the earth's radii, ϕ'_A, ϕ'_B the geocentric latitudes and L_A,L_B the longitudes of stations A and B respectively, we can find the relative rectangular coordinates in the right ascension-declination system of station B with respect to station A at sidereal time zero at station A. Note: as $\theta_A = 0$ the x axis goes through the meridian of station A.

$$\xi_{AB_o} = R_B \cos \phi'_B \cos (L_B - L_A) - R_A \cos \phi'_A$$
$$\eta_{AB_o} = R_B \cos \phi'_B \sin (L_B - L_A)$$
$$\zeta_{AB_o} = R_B \sin \phi'_B - R_A \sin \phi'_A \qquad (29)$$

and obviously

$$R_{AB_o}{}^2 = \xi_{AB_o}{}^2 + \eta_{AB_o}{}^2 + \zeta_{AB_o}{}^2 = R_{AB}{}^2$$

Next we must compute the relative coordinates of station B with respect to A at the time of the event θ_A, the sidereal time of the event at station A. In general $\theta_A \neq \theta_B$. At this time the system will appear as in Fig. 7b. Thus

$$\xi_{AB} = \xi_{AB_o} \cos \theta_A - \eta_{AB_o} \sin \theta_A$$
$$\eta_{AB} = \eta_{AB_o} \cos \theta_A + \xi_{AB_o} \sin \theta_A$$
$$\zeta_{AB} = \zeta_{AB_o} \qquad (30)$$

We also make use of the direction cosines of the zenithal points for the two stations. From Fig. 7b

$$\lambda_{ZA} = \cos \phi_A \cos \theta_A \qquad \lambda_{ZB} = \cos \phi_B \cos \theta_B$$
$$\mu_{ZA} = \cos \phi_A \sin \theta_A \qquad \mu_{ZB} = \cos \phi_B \sin \theta_B$$
$$\nu_{ZA} = \sin \phi_A \qquad \nu_{ZB} = \sin \phi_B \qquad (31)$$

where ϕ is now the geographic latitude.

J. The Radiant

The radiant of a meteor or other moving body is defined as the direction from which it appears to have originated. In our case this point is easily seen to be the point at which the projected great circles defined by the trails on the two plates intersect. The angle between the two great circles is the same as the angle between their poles; the radiant is the cross product of their poles. Hence, using equation 27 for stations A and B

$$\lambda_R \sin Q = \mu_{AP}\,\nu_{BP} - \nu_{AP}\,\mu_{BP}$$
$$\mu_R \sin Q = \nu_{AP}\,\lambda_{BP} - \lambda_{AP}\,\nu_{BP}$$
$$\nu_R \sin Q = \lambda_{AP}\,\mu_{BP} - \mu_{AP}\,\lambda_{BP} \qquad (32)$$

where Q is the angle of intersection between the two meteor trails as seen from station A and station B. The radiant chosen must lie above the horizon; this is done by requiring the dot product $\bar{Z}\cdot\bar{R}$ to be positive or zero, ($\bar{Z}$ being a vector in the zenith direction).

K. The Computation of Range

Since the trajectory of the re-entering vehicle is assumed to lie on a great circle only one possible plane exists through the re-entry track and the observing station; the pole for that great circle is perpendicular to this plane. The distance s_A from the first station (A) to the plane determined by the missile and the second station (B) is seen to be (Fig. 8) the projection of R_{AB} (equation 29) onto a perpendicular to the plane, i.e., onto the line through the pole. Hence, using the direction cosines of the pole (equation 27) we get the distance

$$S_A = \xi_{AB}\,\lambda_{BP} + \eta_{AB}\,\mu_{BP} + \zeta_{AB}\,\nu_{BP} \qquad (33)$$

The distance or range R_{Ai} of station A to any point on the plane is given by $R_{Ai} = S_A \sec \epsilon_i$ where ϵ_i is the angle between $\bar{P}_B$ and the range vector $\bar{R}_{Ai}$. Since points on the re-entering track are just specific points on this plane we have the equation

$$\cos \epsilon_i = \lambda_{Ai} \lambda_{BP} + \mu_{Ai} \mu_{BP} + \nu_{Ai} \nu_{BP}$$

so

$$R_{Ai} = S_A / \cos \epsilon_i \qquad (34)$$

L. The Computation of Height

In computing height above sea level, the height h_{Ai} above a plane through the station normal to the zenith direction is first computed. The zenith distance Z_{Ai} for any point on the re-entry is given by

$$\cos Z_{Ai} = \lambda_{ZA} \lambda_{Ai} + \mu_{ZA} \mu_{Ai} + \nu_{ZA} \nu_{Ai} \qquad (35)$$

see Fig. 9. Thus the height above the plane normal to the zenith is found by

$$h_{Ai} = R_{Ai} \cos Z_{Ai} \qquad (36)$$

To obtain height H_i above sea level, the elevation of the station H_A, and a correction δh_{Ai} for the earth's curvature must be added. As in Fig. 9 the total height of the vehicle from the center of the earth is given by

$$(\rho_A + h_{Ai} + \delta h_{Ai})^2 = \rho_A^2 + R_{Ai}^2 + 2\rho_A R_{Ai} \cos (Z_{Ai})$$

and thus solving for δh_{Ai}

$$\delta h_{Ai} = (\rho_A^2 + R_{Ai}^2 + 2\rho_A h_{Ai})^{1/2} - (\rho_A + h_{Ai}) \qquad (37)$$

The total height above mean sea level is

$$H_i = h_{Ai} + H_A + \delta h_{Ai} \qquad (38)$$

where H_A is the elevation of the station.

M. The Calculation of Distance Along the Trail

After the ranges have been computed for any two points on the re-entry track the distance between them can easily be found. The components of the vector R_{Ai} in the astronomical equatorial system are

$$\xi_{Ai} = R_{Ai} \lambda_{Ai}$$

$$\eta_{Ai} = R_{Ai} \mu_{Ai}$$

$$\zeta_{Ai} = R_{Ai} \nu_{Ai} \qquad (39)$$

Letting the first point seen on the trail be called D_{A1} we can find the distance from the beginning of the trail to any other point by

$$D_{Ai} = \left[(\xi_{Ai} - \xi_{A1})^2 + (\eta_{Ai} - \eta_{A1})^2 + (\zeta_{Ai} - \zeta_{A1})^2 \right]^{1/2} \qquad (40)$$

N. The Computation of Relative Time

Except in special cases where a common point can be identified along the trail in both photographs, it is impossible to find exactly simultaneous points on the two plates; also, no absolute time is available for any point. However, if one or more of the photographs has been chopped, a relative time (i.e., time measured from an arbitrary zero point) can be computed and makes possible the calculations of height and range as functions of time and also an estimate of the velocity of the vehicle as the time derivative of distance along the trail.

The cameras used to compute relative time employ a shutter which opens and closes at known constant intervals, thus making chops on the plate. The coordinates of the mid-point of each chop are read; these points correspond to known relative times since the chopping rate is known.

If P is the period of revolution of the rotating shutter and N the number of occultations, all equally spaced, per revolution, the instant t_i corresponding to the i'th break is

$$t_i = i \left(\frac{P}{N}\right) + \Delta t_i \qquad (41)$$

where Δt_i is a function of the position of the re-entry body on the plate relative to the center of rotation of the shutter. For calculations not involving Super-Schmidt cameras it will be assumed that Δt_1 is negligible.

For Super-Schmidt cameras the correction is often fairly large. As a first approximation let us assume that the projection center (X_c, Y_c) very nearly coincides with the center of rotation of the shutter (X_q, Y_q); in practice this approximation is usually valid for a distance between the two centers less than one cm. In Fig. 10 let X_i, Y_i be the coordinates of the i'th point on the trail. The angle that the blade edge makes with the i'th dash is obviously given by

$$\tan \omega_i = (X_i - X_q) / (Y_i - Y_q) \tag{42}$$

If ω_o is the value of ω at the origin of the time scale we have

$$\Delta t_i = \pm (\omega_i - \omega_o) P/2\pi \tag{43}$$

where the + sign is used when the shutter edge chases the re-entry body as in Fig. 10, since here more than one revolution of the shutter occurs between chops and the - sign is used when the edge runs toward the body.

If the distance between the center of rotation of the shutter and the projection center is too large for equation 42 to be valid an exact expression for $\tan \omega_i$ has been found by Whipple and Jacchia[1] to be

$$\tan \omega_i = z \left[u (X_i - X_q) - (Y_i - Y_q) \right] / \left[u (Y_i - Y_q) + (X_i - X_q) \right] \tag{44}$$

where

$$u = (Y_q - Y_c) / (X_q - X_c)$$

$$z^2 = 1 + (1/f^2) \left[(X_q - X_c)^2 + (Y_q - Y_c)^2 \right]$$

and f is the focal length of the camera.

O. The Problem of Stationary Cameras

One or both of the photographs used in the analysis may have been made by a stationary camera. In this case the star image consists of a trail rather than a point on the plate. The ends of this trail correspond to the opening and closing of the camera. Thus the end of the star trail where the measurement is made is not simultaneous with the time of the event. The declinations of the stars are unchanged but the right ascensions are shifted by an amount $\Delta\theta = \theta_A - \theta'_A$ where θ_A is the sidereal time of the event and θ'_A the time at the end of the star trail where the plate is measured. Therefore the direction cosines $(\lambda'_{Ai}, \mu'_{Ai}, \nu'_{Ai})$ computed by equation 4 must be changed to the true direction cosines by

$$\lambda_{Ai} = \lambda'_{Ai} \cos \Delta\theta - \mu'_{Ai} \sin \Delta\theta$$

$$\mu_{Ai} = \mu'_{Ai} \cos \Delta\theta + \lambda'_{Ai} \sin \Delta\theta$$

$$\nu_{Ai} = \nu'_{Ai} \tag{45}$$

It is easily shown that if only the direction cosines of the plate center and of the $\bar{\xi}$ and $\bar{\eta}$ axes are corrected by equation 45 from the time of the star trail's end to the time of the event, the direction cosines of each point on the re-entry trail computed from equation 24 will be correct.

III. THE EXPERIMENTAL OPTICAL PROGRAM

The optical reduction method outlined in the present paper for determining the space trajectory of a re-entering body requires that the event be photographed simultaneously from two stations separated by a relatively long baseline. If velocity is to be computed, at least one of the cameras must be equipped with a chopping shutter so that relative time may be obtained. The Trailblazer program utilizes two Baker Super-Schmidt cameras, one at Arbuckle Neck, the other at Eastville. In addition, photographs taken by NASA on ballistic cameras from their Wallops Island and Coquina Beach stations are usually available. Figure 11 shows the relation of these camera sites to each other and to the launch site.

A. The Super-Schmidt Camera

Figure 12a is a photograph of a Baker Super-Schmidt camera used to obtain the re-entry photographs. This camera has an aperture of 12.25 inches, a focal length of 8 inches, an effective focal ratio of 0.85 and a field of view of approximately 58 degrees. It is supplied with an interchangeable shutter and motor capability, so that, depending upon the number of blades in the shutter and the rpm of the motor, different chopping speeds are possible. The surface of the shutter is immediately in front of the photographic emulsion.

A schematic diagram of the optical system appears in Fig. 12b. Two part-hemispherical correcting shells, concentric with the center of curvature of the spherical mirror, provide almost complete correction of spherical aberration. Any remaining aberrations and achromatism over the range 3800-7000 Å are corrected for by an aspherical correcting plate through the center of curvature. The spherical focal surface, concentric with the rest of the system, lies just within the second shell. A light ray entering the system passes first through the outer shell, the correcting plate, and the second shell before it is reflected at the mirror; it then passes through the inner shell a second time before it reaches the focal surface.

Since the focal surface of the Super-Schmidt is spherical, flat film must be molded to this shape. The film is held in the focal plane, convex toward the mirror, by a partial vacuum (about 20" Hg) between the film and the focal plane. A special, very fast, blue sensitive X-ray film is usually employed in these tests. It is made by Eastman Kodak especially for the Harvard Meteor Project.

The optical projection in the Super-Schmidt camera is very nearly a truly concentric spherical projection; this allows the star images to maintain their relative positions on the celestial sphere without distortion. The curved film is projected onto a flat plate by a very nearly gnomonic projection; this flat plate (a positive) is then used for the analysis.

Although much of our work is done with the Super-Schmidt cameras the reduction methods and the computation program discussed here will work with any set of ballistic plates.

B. Star Identification

At each station the re-entry trail and the field stars are photographed simultaneously. If the photograph is taken by a camera equipped with an equatorial mount, such as the Super-Schmidt, the stars appear as points on the plate (Fig. 13a); if the camera is stationary, the stars will appear as streaks (Fig. 13b). However, if the time of opening and closing of the shutter is known the ends of these streaks are identifiable in time and may be used in the analysis as discussed in Section II-O.

The approximate coordinates of the plate center in right ascension and declination are recorded by the camera operator; the direction of north on the photograph is also indicated. Thus the general position and orientation of the photograph are known. For Super-Schmidt plates, in particular, the use of Norton's Star Atlas for a preliminary identification of the brighter stars is recommended since the scale of its star maps is very nearly that of the Super-Schmidt film. By overlaying the film on the charts, many of the stars may be immediately identified and the same stars easily found on the BD or CD chart, thus outlining the area of interest. The position of the trail among the stars is then found on the BD or CD charts and may be sketched in lightly for future reference.

Stars to be measured should be chosen at uniform intervals along the trail, (X-direction) and alternately on opposite sides of the trail (+ and - Y). In order to minimize centering errors the stars chosen should be of nearly the same photographic magnitudes and as near the plate limit as possible. The stars to be measured may be marked on the star charts and their BD or CD numbers as well as their approximate coordinates recorded from the BD or CD tables. When the BD or CD number of a star is known its position may be found in the more exact star catalogues.* The following information should be recorded for each star:

Right Ascension

Declination

Equinox of Reference

Epoch of Observation

Components of Proper Motion in Right Ascension and Declination.

The stars to be measured may be marked on the glass side of the plate. About 8 to 10 stars are usually sufficient for a plate calibration; at least 6 should always be used.

C. The Position of the Plate Center

In addition to the coordinates of the selected stars, the coordinates of the plate center must be estimated. Four stars are chosen such that two lines intersecting at the center may be drawn through the stars (i.e., each two stars will lie on a line through the center; the stars should be on opposite sides of the center). The four stars are identified on the star charts and the lines drawn lightly on the charts. The point of intersection of the two lines on the star chart may be read off as the approximate right

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*A list of star catalogues will be found in Appendix A.

ascension and declination of the plate center.

D. The Measurement of the Plates

The plates should be measured on a measuring engine equipped with two mutually perpendicular precision screws so that the X and Y coordinates may be measured simultaneously in a plane (Fig. 14). The plate should be oriented in the measuring engine so that the trail of the re-entry is parallel to the X axis; if the trail is not completely straight, the X axis should be made parallel to the straight line portion. All measurements of the plate must be made without removing the plate from the machine. To avoid possible backlash effects, all measurements are made with the screw turning in the same direction (customarily toward increasing scale readings). To check the consistency of the readings over a period of time, two points are selected near the ends of the trail and read at occasional intervals during the measuring of the plates. In this way any disturbances of the machine by temperature changes or mechanical vibration will show up. Readings are made to the nearest micron.

Various methods of finding the coordinate of a point are possible -- usually an average of three settings is sufficient, as long as the readings do not vary by more than a few microns. It is sometimes advisable to repeat a series of measurements with the plane rotated 180° to check for possible systematic or accidental errors. Whatever method of averaging is used, only one value of each point is used in the program. Therefore any averaging or statistical analysis on the readings must be performed prior to entry into the program.

When the plate has been adjusted in the machine so that the X-axis is parallel to the re-entry trail, the center point of each star selected for analysis is read to get the coordinates in X and Y of the star. Next, the trail, starting from the first distinct point and ending at the last distinct point (whether or not these points are seen as dashes) is divided up into from about 10 to 20 equal parts (in X). The Y measure at each of these equally spaced X's is read for computing the trail equation.

If the re-entry event is photographed with a camera equipped with a chopping shutter the re-entry trail will appear as a fairly straight dashed line -- the dashes being caused by the shutter breaks (Figs. 13a and 13b). If the camera does not have a chopping shutter, the re-entry trail will appear as a streak among the stars (Fig. 13c). The X-measurement of the middle of each dash should be taken in the case of a chopped plate. The Y coordinate may then be computed for each dash by the trail equation so that Y need not be read for the dashes. On a streak photograph any point of special interest, such as a flare, should be measured. Also, the very first and very last points visible on the trail must be measured. At least five measurements should always be made along the trail.

A diagramatic sketch of the required measurements in shown in Fig. 15. This diagram is only theoretical and does not represent a particular re-entry. Stars appear over the entire plate, but only those near the trail are of interest here. Stars labeled A-H have been selected for measurement. These stars are seen to be selected on alternate sides of the trail and relatively evenly spaced along the trail. Star F is seen to lie on the trail itself. The actual trail lies between the two lines marked T.B. and T.E. (beginning and end); it will be noticed that many dashes are missing at the start of the trail and a few near the end. The measurements for the trail equation will be made at equal X increments between TDB and TDE, the first and last distinct points. In this case the interval is divided into 10 equal increments (in X) labeled T1, T2...T11. Notice that the reading is taken along the trail and that the point read may lie between two dashes. A Y measure is taken for each of the 11 X's. The first point T.B. and the last point T.E. are also measured.

The center of each dash should next be read in X starting with the first distinct dash, D1 and continuing for the rest of the distinct dashes (one or more may be missing as is dash 18 in the diagram). The flare occurring at dash 17 is particularly interesting since it may also show on the other plate and so could be used as a common point. The plate center should also be measured in X and Y.

E. Computations

After the measurements have been made, the next step is a plate calibration which will relate the X, Y measurements on the plate to positions in the sky. The plate calibration program is run using as input the measured X, Y of the known stars and the coordinates of the stars from the catalogues. The output consists of inverse plate constants, and direction cosines for the plate center and the coordinate axes which are needed as input to the optical trajectory program. When the results of the plate calibration program are available, the residuals (or Δ X and Δ Y) for each star are examined; if any of them are extremely large, the calibration is repeated omitting the offending star. When the calibration is satisfactory, i.e., the residuals are very small, the residuals may be plotted against X (Fig. 16). If the residuals are within the limits of measurement they are usually ignored; in other cases they are applied to each measurement along the trail as discussed in Section II-F.

When both plates have been calibrated by the plate calibration program, the input cards to the optical trajectory program are made as discussed in Section VI. The optical trajectory program uses the results of the calibration program, the geographic coordinates of the stations, the time of the event, data about the cameras, the measured X, Y readings taken at equal X increments for computing the trail equation and the X readings for each measured point along the trail from the two plates and has as output such meaningful parameters as range, height, direction cosines, and distance along the trail for each measured point along the trail.

F. The Reduction of Trailblazer Ik

To demonstrate the use of the optical trajectory program the reduction of Trailblazer Ik will be discussed. Trailblazer Ik was fired from Wallops Island on 27 July 1962 at 21:50:00 E.S.T. and was photographed by the Super-Schmidt cameras at Arbuckle Neck and at Eastville at 21:56:21.583 (Figs. 17a and 17b). The plates were read at Harvard University on a Mann Measuring Engine as discussed above.

The astronomic coordinates and the X, Y measurements of the stars are given in Table II for the SL (Arbuckle Neck) camera. The coordinates have already been corrected for proper motion. Care must be taken in using the values of proper motion given in the star catalogues, since some catalogues give proper motion as $\Delta\alpha$ and some as $\Delta\alpha \cos\delta$ per year or per century. Corrections in proper motion are made to the nearest month. Also, the X, Y measures have been averaged to give one reading for each point.

TABLE II

Stars Identified and Measured for the SL Photograph of Trailblazer IK taken at Arbuckle Neck

Name	Right Ascension		Declination		Equinox	X mm	Y mm
A	20h 6m	13.754s	-28° 35'	13.95"	1950	173.1015	20.411
B	20h 7m	28.968s	-29° 54'	35.97"	1950	178.155	20.738
C	20h 8m	17.23 s	-30° 56'	52.6 "	1950	182.086	21.1255
D	20h 13m	45.23 s	-32° 45'	45.2 "	1950	189.9875	18.8385
E	20h 14m	25.78 s	-33° 50'	22.2 "	1950	194.171	19.4495
F	20h 17m	2.57 s	-34° 44'	34.3 "	1950	198.177	18.4505
Plate Center	19h 45m		-18° 17'		1855	133.205	23.520

The first step is to perform a calibration of the plate. (We will discuss only the reduction of the SL data as the SS data are reduced in an identical manner.) From Table II we see that we have a total of 6 stars all of which have their coordinates referred to the equinox of 1950. We also see that the coordinates of the plate center are referred to the equinox of 1855. Thus two sets of precession constants must be used; the first will precess the coordinates of the plate center from 1855-1963 and the second will precess the coordinates of the stars from 1950-1963 (the nearest 1 January). The correct precession constants are shown in Table V. $\Delta\theta$ will be zero since the Super-Schmidt cameras track the stars. We will run one four constant solution, using stars A and F (or 1 and 6). The card set-up and a sample computer print-out are given in Appendix B.

Upon inspection of the results of the calibration program, it is seen that the residuals are very small and within the accuracy of measurement. Therefore, no corrections need be applied to the measured values of X and Y. The six constant solution will be used in the reduction. The next step is to prepare the cards for the Optical Trajectory Program.

The event took place at 21:56:21.583 E.S.T. on 27 July 1962. Converting to Greenwich time, we have 02:56:21.583 G.M.T. on 28 July 1962. We must therefore find the θ_o for 28 July in the 1962 Almanac since the date at Greenwich must be used. It is found to be 20h 20m 43.038s. The Greenwich time and geographic coordinates are given on the next card.

From Table III we see that 15 equally spaced points were read along the trail. The first X point is at 176.000mm and an average Y on the trail is about 20.140mm. These are subtracted from each reading along the trail and the differences are inserted into the program. This set of cards will be used to compute the trail equation to give Y as a function of X.

The next two cards will give the inverse plate constants (from Page 4 of our print-out for the six constant solution) and the next three will give direction cosines of the plate center and standard axes (from Page 5 of the print-out). The names of the variables in the plate calibration print-out are identical to those used in the card write-up (Section V).

The next card gives the X coordinates of 5 points of special interest (the first point must be the physically first point read on the trail, T.B. in our diagram and the last point must be the physically last point, T.E.). The other three points may be any intermediate points such as flare points or other points of special interest.

The next cards will give information about the dashes as shown in Table IV. Table IV is not complete but enough dashes are shown to give an understanding of the card set up.

TABLE III

X, Y Readings at Equal X Increments Used in Computing the Trail Equation Data from the SL Photograph of Trailblazer Ik Taken at Arbuckle Neck

X	Y	ΔX	ΔY
mm	mm	mm	mm
176.0	20.140	0	0.000
177.5	20.141	1.5	0.001
179.0	20.148	3.0	0.008
180.5	20.142	4.5	0.002
182.0	20.146	6.0	0.006
183.5	20.142	7.5	0.002
185.0	20.139	9.0	-0.001
186.5	20.140	10.5	0.000
188.0	20.144	12.0	0.004
189.5	20.142	13.5	0.002
191.0	20.140	15.0	0.000
192.5	20.139	16.5	-0.001
194.0	20.137	18.0	-0.003
195.5	20.140	19.5	0.000
197.0	20.136	21.0	-0.004

End Point X = 175.861
Y = 198.644

TABLE IV

X Readings for Dashes for the SL Photograph of Trailblazer Ik taken at Arbuckle Neck

Dash #	Wt.	X
		mm
1	4	175.946
2	4	176.245
3	1	176.534
4	2	176.796
5	1	177.057
6	2	177.332
87	1	198.567

TABLE V

Precession Constants

	1855-1963	1950-1963
X_x	0.99965376	0.99999498
X_y	-0.02412862	-0.00290553
X_z	-0.01049490	-0.00126316
Y_x	0.02412862	0.00290553
Y_y	0.99970885	0.99999578
Y_z	-0.00012664	-0.00000183
Z_x	0.01049490	0.00126316
Z_y	-0.00012661	-0.00000183
Z_z	0.99994490	0.99999920

Table VI gives information about the cameras and is used in making out the remaining cards. A similar set of cards must be made for the SS camera. Appendix C gives a listing of cards used as input to the trajectory program, and a sample computer output. Not all of the card set up and print-out are shown, since Appendix C is meant to show a sample calculation and not give useful values of a particular shot.

TABLE VI

Camera Data for Trailblazer Ik

	SL Camera	SS Camera
XQ	133.205	126.656
YQ	23.520	10.333
XC	133.205	126.656
YC	23.500	10.333
F^2	4074	4074
Period	0.1	0.1
Occult	2	2
Direction	Clockwise	Counter Clockwise

IV. THE PLATE CALIBRATION PROGRAM

The plate calibration program is designed to carry out the computations necessary to relate X, Y measurements on the photographic plate to astronomical coordinates and to compute residuals for the correction of the measurements of points along the trail. The plate constants are computed by both the four and the six constant solutions and the method giving the best (i.e., smallest or most consistent) residuals is used in the later work.

The input to the program includes the right ascension and declination and the X, Y measurements of each star selected and of the plate center. The astronomical coordinates of the stars are corrected for proper motion to the nearest month before being inserted into the program; however, the corrections for precession to the nearest 1 January are performed within the program in direction cosine form. A generalized flow chart of the plate calibration program and of subroutine SIXCON appears in Fig. 18.

The computer output consists of the following print-outs:

Page 1 Listing of the input data for the plate center and each star in easily understandable form

Page 2 Direction cosines for the uncorrected ($\Delta\theta = 0$) plate center and the coordinate axes
Direction cosines and standard coordinates (σ, $\overline{\eta}$, $\overline{\xi}$) for each star

Page 3* Plate constants, inverse plate constants, and residuals for the four constant solution (each four constant solution will be on a separate page)

Page 4* Plate constants and inverse plate constants and residuals for the six constant solution

Page 5* Corrected direction cosines for the plate center and coordinate axes (Note: since in our case $\Delta\theta = 0$, the direction cosines on Page 5 are identical with those on Page 2).

Appendix B gives a print-out sample of the results of the plate calibration program.

The residuals should be inspected to see if any star gives either very large (probably above 100 microns) or very inconsistent values. Such a star should be eliminated and the calibration program rerun. When the residuals seem to be small and lie on a smooth curve (to within the accuracy of measurement), they should be plotted against the X reading.

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*If more than one four constant solution is run, they will appear on pages 3', 3", etc.

Fig. 16 is a sample curve of the residuals for a theoretical case. A smooth curve is drawn through the residuals and this curve (one for ΔX and another for ΔY) is used to correct the sets of X, Y readings to be used in computing the trail equation and the set of X readings measured at points to be reduced (equation 23').

A listing of the plate calibration program appears in Appendix D. The six constant solution is performed by subroutine SIXCON which solves the matrix equation

$$A\,X = B$$

where

$$A = \begin{pmatrix} \Sigma\,\bar{\xi}_i^{\,2} & \Sigma\,\bar{\xi}_i\bar{\eta}_i & \Sigma\,\bar{\xi}_i \\ \Sigma\,\bar{\xi}_i\bar{\eta}_i & \Sigma\,\bar{\eta}_i^{\,2} & \Sigma\,\bar{\eta}_i \\ \Sigma\,\bar{\xi}_i & \Sigma\,\bar{\eta}_i & N \end{pmatrix}$$

$$X = \begin{pmatrix} a_x & a_y \\ b_x & b_y \\ c_x & c_y \end{pmatrix}$$

$$B = \begin{pmatrix} \Sigma\,x_i\bar{\xi}_i & \Sigma\,y_i\bar{\xi}_i \\ \Sigma\,x_i\bar{\eta}_i & \Sigma\,y_i\bar{\eta}_i \\ \Sigma\,x_i & \Sigma\,y_i \end{pmatrix} \qquad (46)$$

using library routine XSIMEQF. This routine solves equation 46 for X by triangularization of the A matrix. If the solution is successful, the X matrix will replace the original A matrix in storage. In the program the word MATRIX is a code which equals 1 for a successful solution of the equation, equals 2 for overflow or underflow, and equals 3 for a singular matrix. The arguments used in calling the subroutine XSIMEQF together with their values for this example are:

Arg_1 = maximum value of subscript I of A (I, J) = 10

Arg_2 = number of rows in matrix A = 3

Arg_3 = number of columns in matrix B = 2

Arg_4 = name of matrix A = A

Arg_5 = name of matrix B = B

Arg_6 = scale factor for use in calculation = 1

Arg_7 = one dimensional erasable array for use in solution = TWO = 10

A more detailed description of the subroutine XSIMEQF is available.

The data cards are set up as follows:

Data Cards for the Plate Calibration Program

Card 1 Column 1 contains a "1" and columns 2-72 contain a Hollerith title to be written on tape A3. This title may contain any identifying information desired.

Card 2 Contains two fixed point numbers (format 2I5)

NN = number of sets of precession constants used in the reduction in columns 1-5 (format I5) Up to 5 sets may be used

N = number of stars used in the reduction in columns 6-10 (format I5).

The next group of cards will consist of 4 cards for each of the NN sets of precession constants.

The first card of each precession set will be a Hollerith date card constructed as follows:

A zero in column 1

Year of equinox of table in columns 2-5

Hollerith hyphen in column 6

Year of event to 1 January in columns 7-10.

The next 3 cards of the precession set will each have 3 precession constants in floating point form (format 3E15.8) as follows:

Precession card 2

X_x in columns 1-15

X_y in columns 16-30

X_z in columns 31-45

Precession card 3

Y_x in columns 1-15

Y_y in columns 16-30

Y_z in columns 31-45

Precession card 4

Z_x in columns 1-15

Z_y in columns 16-30

Z_z in columns 31-45

Each precession set will also have a number J associated with it; J being assigned according to the placement of the set in the deck, i.e., J = 1 for the first precession set read in, J = 2 for the second precession set, etc. The precession set to be used in each case will be that one corresponding to the equinox for which the coordinates of the star (or plate center) are recorded in the standard tables.

The next card will have 10 numbers (formats 2F4.0, F8.4, 2F4.0, F7.3, 2F10.5, I5, E15.8) which give the coordinates of the plate center as follows:

Right ascension in hours	in columns 1-4
Right ascension in minutes	in columns 5-8
Right ascension in seconds	in columns 9-16
Declination in degrees	in columns 17-20
Declination in minutes	in columns 21-24
Declination in seconds	in columns 25-31
X reading of plate center in mm	in columns 32-41
Y reading of plate center in mm	in columns 42-51
J = number of precession set to use	in columns 52-56
$\Delta\theta$ = correction of plate center for stationary cameras	in columns 57-71

(Note: $\Delta\theta = 0$ for tracking cameras.)

The next N cards will give the coordinates of the N stars to be used in the analysis and will have 9 numbers per card (formats 2F4.0, F8.4, 2F4.0, F7.3, 2F10.5, I5). Corrections in right ascension and declination for proper motion must be made before the values are punched on the cards. The N cards will have the following format:

Right ascension in hours	columns 1-4
Right ascension in minutes	columns 5-8
Right ascension in seconds	columns 9-16
Declination in degrees	columns 17-20
Declination in minutes	columns 21-24
Declination in seconds	columns 25-31
X reading of plate in mm	columns 32-41
Y reading of plate in mm	columns 42-51
J = number of precession set to use	columns 52-56

Each star will have a number L associated with it depending on its placement in the deck, i.e., L = 1 for the first star card read in, L = 2 for the second card, to L = N for the last star card read in.

The next card will have one number MM (format I5) in columns 1-5; MM is the number of four constant solutions to be run. (Note: it may be desired to try different combinations of stars as end points in the four constant solution; this card allows for this possibility.)

The last MM cards will each have 2 numbers (format 2I5), the first number is the L for the star used as the beginning point in the four constant solution in columns 1-5 and the second number is the L for the star to be used as end point in columns 6-10.

V. THE OPTICAL TRAJECTORY PROGRAM

The optical trajectory program computes such physically meaningful parameters as range, height, direction cosines, distance along the trail, and relative time for the X, Y measurements of two photographic plates of the re-entry event. Cards may be made for a future velocity calculation; however, velocity is not computed by the present program.

Input to the program includes the sets of X, Y readings taken at equal X increments along the trail to be used to compute the trail equation, the sets of X readings taken at selected intervals along the trail (such as the centers of the dashes) where a reduction is desired, the time of the event, the geographic coordinates of the camera stations, the plate calibration data obtained from running the calibration program for the two plates, and information about the cameras used to photograph the event. Only one position measurement is entered for each point read; any averaging or residual corrections necessary to obtain this measurement is done prior to entry into the program.

A generalized flow chart of the optical trajectory program and of each subroutine appears in Fig. 19. The program itself is given in Appendix E.

The computer output consists of the following print outs:

1. A listing of the input data
2. Time (Greenwich and sidereal) and geographic coordinates of the two stations; direction cosines for vector from station 1 to station 2 at sidereal time zero and at the time of the event
3. Plate calibration data, trail equation, X, Y, and direction cosines of the five points and direction cosines of the pole for each station
4. Direction cosines of the radiant
5. Ranges and heights for the five points
6. Tables of X, Y, range, cos Z, and height above mean sea level for each dash for the two stations
7. Tables of relative time, range, height, and direction cosines in the altitude-azimuth system for each dash for the two stations

Note: Certain checks have been performed in the program and the small errors found printed out for inspection. For example, checks have been made to determine if the sum of the squares of the direction cosines of a point equals one.

A sample output is shown in Appendix C for the Trailblazer Ik re-entry.

A card output containing values of distance along the trail, relative time, range, height, direction cosines in the altitude-azimuth system, dash number and dash weight is also available.

The dash weight gives a measure of the confidence level that the person reading the plate put on that dash. The weights are assigned as follows:

1	very poor
2	poor
4	fair
6	good
8	excellent

The weights are not used in the calculations of this program but are included in the card output for possible use in future velocity calculations.

The following is a description of the input cards necessary for running the optical trajectory program.

Card 1 Column 1 contains a "1" and columns 2-72 contain a Hollerith title to be written on tape A3. This title may be any identifying information desired.

Card 2 will contain three numbers (formats 2F4.0, F8.4)

Theta0	in hours	in columns 1-4
Theta0	in minutes	in columns 5-8
Theta0	in seconds	in columns 9-16

Theta0 is the sidereal time at Greenwich for 0h Universal Time for the day of the event at Greenwich (the day will be defined to start at 0000 Universal Time); the theta0 is found in the table of "Universal and Sidereal Times" of <u>The American Ephemeris and Nautical Almanac</u> for the proper year - theta0 is tabulated separately for each day of the year.

Following cards 1 and 2 which give general information about the event, we will have two sets of cards, one for each station. In the following discussion the making of these cards for station A will be shown; those for station B are made in exactly the same way and immediately follow those of station A in the deck.

Card 3 will have a zero in column 1 and any Hollerith identification of station A desired in columns 2-72.

Card 4 will have 10 numbers giving the coordinates of station A (format 3(2F4.0, F8.4), F11.7)

Event time (Greenwich)	in hours	in columns 1-4
Event time	in minutes	in columns 5-8
Event time	in seconds	in columns 9-16
Latitude of Station A	in degrees	in columns 17-20
Latitude	in minutes	in columns 21-24
Latitude	in seconds	in columns 25-32
Longitude of Station A	in hours	in columns 33-36
Longitude	in minutes	in columns 37-40
Longitude	in seconds	in columns 41-48
Elevation of Station A	in kilofeet	in columns 49-59

The next group of cards will be used to compute the trail equation by the least squares method. A starting X_o and Y_o are chosen and the calculation performed on the differences from these values where

$$\Delta X = X_i - X_o$$

$$\Delta Y = Y_i - Y_o.$$

The X_o chosen is the first X reading on the trail, the Y_o is about the average of the Y readings along the trail.

Card 5 contains a single number N which is the number of points along the trail (format I5) in columns 1-5.

Card 6 contains two numbers (format 2 E15.8)

X_o	in columns 1-15
Y_o	in columns 16-30.

Card 7 etc. The next N/2 or (N/2) + 1 cards contain the values of ΔX_i and ΔY_i punched two to a card (format 4 E15.8)

ΔX_i	in columns 1-15
ΔY_i	in columns 16-30
ΔX_{i+1}	in columns 31-45
ΔY_{i+1}	in columns 46-60

The next five cards give calibration data obtained as output from the plate calibration program; each card (except the last which has only two numbers) has three numbers (format 3 E15.8). These values are given as output from the plate calibration program where they are labeled by the same symbol as in this description.

Card 1 of the 5

AEXI	in columns 1-15
BEXI	in columns 16-30
CEXI	in columns 31-45

Card 2 of the 5

AETA	in columns 1-15
BETA	in columns 16-30
CETA	in columns 31-45

Card 3 of the 5

LAMC	in columns 1-15
LAMEXI	in columns 16-30
LAMETA	in columns 31-45

Card 4 of the 5

MUC	in columns 1-15
MUEXI	in columns 16-30
MUETA	in columns 31-45

Card 5 of the 5

NUC	in columns 1-15
NUETA	in columns 16-30

The next card gives 5 values of X; the first X on this card must be the X reading at the physically first point of the trail and the fifth X on this card must be X reading at the physically last point of the trail. The other three X's should be common points if any exist or any other particularly interesting value of X. Note: this card must always contain 5 X's whether or not any common points are found; also, the first and last X on the card must be the physically first and last X read on the event (format 5F10.5).

The next card will contain one number (format I5) which will be the number of dashes read along the trail plus 2 (for the end points of the trail which in general are not the middle of dashes)

NUM = number of dashes + 2 in columns 1-5

The next group of cards will give the information for each dash with three readings to a card. The first reading will correspond to the beginning of the trail and the last to the physically last point on the trail (these two readings will not, in general, be read from the middle of the dashes). These two end points will have a dash number of 0 and a weight of 1 (format 3 (2I5 , F10.5)).

Dash number	in columns 1-5
Weight of dash	in columns 6-10
X reading of dash	in columns 11-20
Dash number	in columns 21-25
Weight of dash	in columns 26-30
X reading of dash	in columns 31-40
Dash number	in columns 41-45
Weight of dash	in columns 46-50
X reading of dash	in columns 51-60

The next and last group of the set gives information about the cameras used.

Card 1 of the group contains four numbers (format 4 E15.8)

Observed X coordinate of shutter center XQ	in columns 1-15
Observed Y coordinate of shutter center YQ	in columns 16-30
X projection center XC	in columns 31-45
Y projection center YC	in columns 46-60

The next card will contain 3 numbers (format 3 E15.8)

F^2 = focal length squared	in columns 1-15
Period of revolution of shutter	in columns 16-30
Number of occultations per revolution	in columns 31-45

The last card for station A will have two numbers (format 2I5)
The first number will be in columns 1-5

= 1 if card output is desired

= 0 if cards are not desired

The second number will be in columns 6-10

= 1 if counterclockwise revolution of Super-Schmidt camera

= -1 if clockwise revolution of Super-Schmidt camera

= 0 if non-Super-Schmidt camera

This completes the input cards for station A. A second set in the same formats starting with card 3 will be required for station B.

ACKNOWLEDGEMENT

The author wishes to express her appreciation to Dr. R. E. McCrosky, Mrs. Annette Posen and Mr. G. Davoren of the Harvard Observatory who supplied training in the astrometric techniques discussed in this paper and who supplied a copy of the original Harvard Meteor Program and to Mr. E. Weston and Mr. J. Knight who supplied the list of star catalogs and charts for Appendix A. She especially wishes to thank Dr. H. L. Kasnitz and Mr. G. M. Shannon of Lincoln Laboratory who reviewed the paper and offered many helpful suggestions.

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4. W. M. Smart, Text Book on Spherical Astronomy, Cambridge University Press (1960).

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3-21-6094

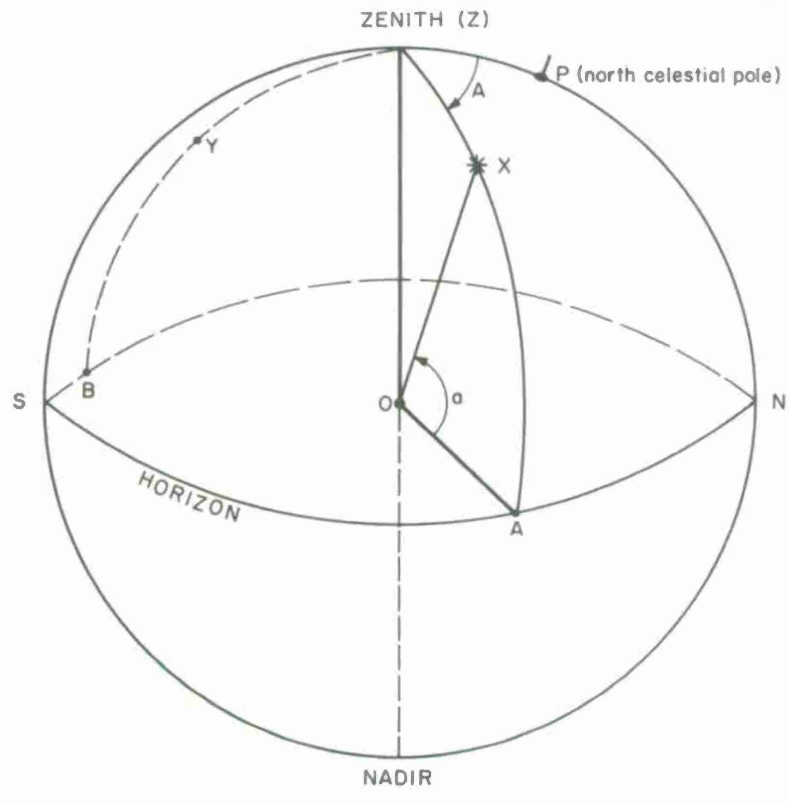

Fig. 1a. The altitude-azimuth system.

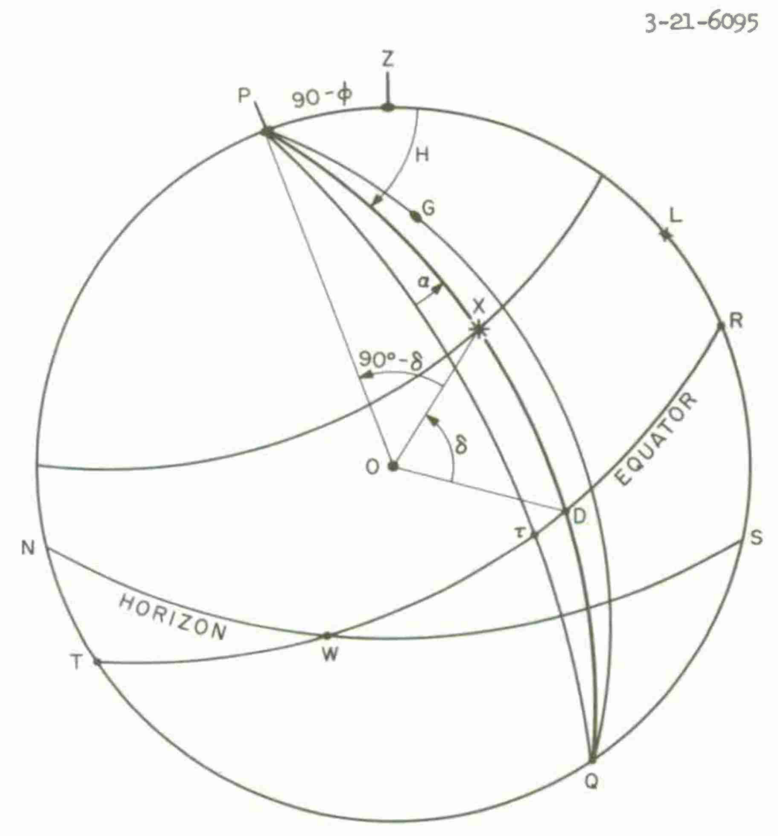

Fig. 1b. The declination-hour angle and the declination-right ascension systems.

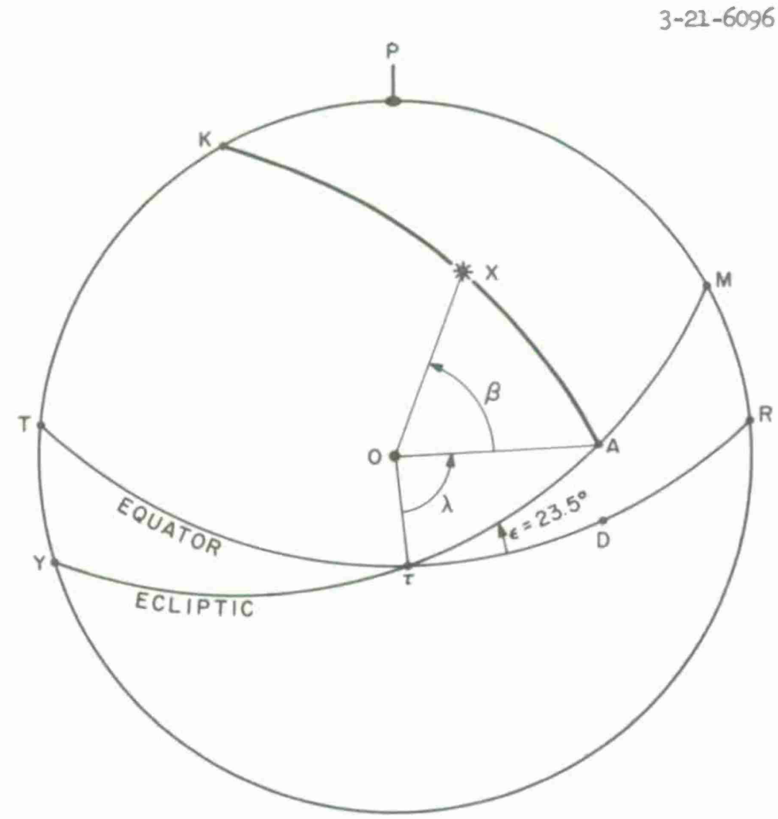

Fig. 1c. The celestial latitude-longitude system.

3-21-6097

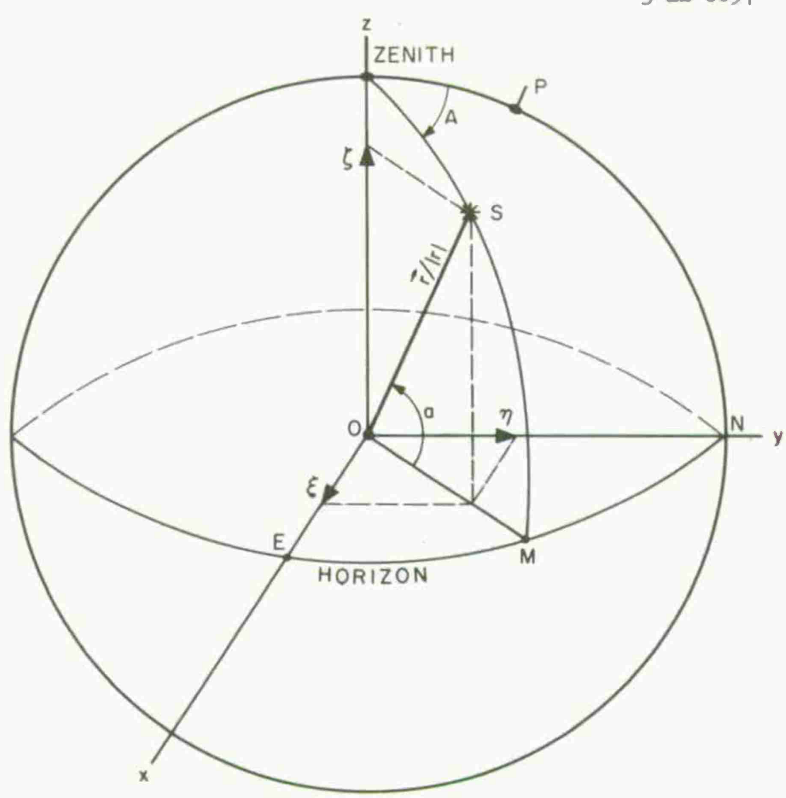

Fig. 2a. The altitude-azimuth system.

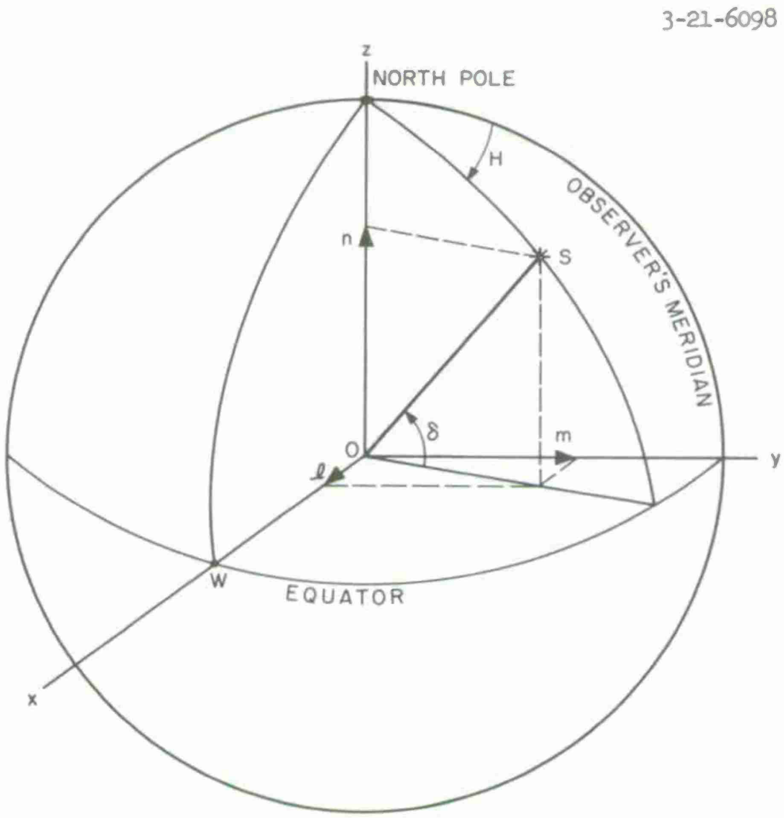

Fig. 2b. The declination-hour angle system.

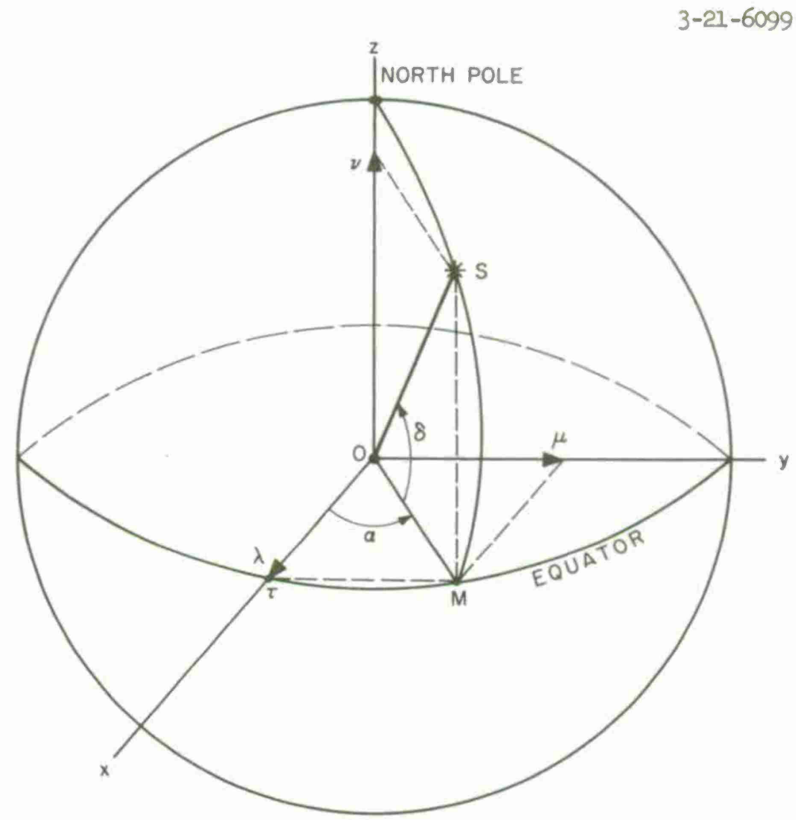

Fig. 2c. The declination-right ascension system.

3-21-6100

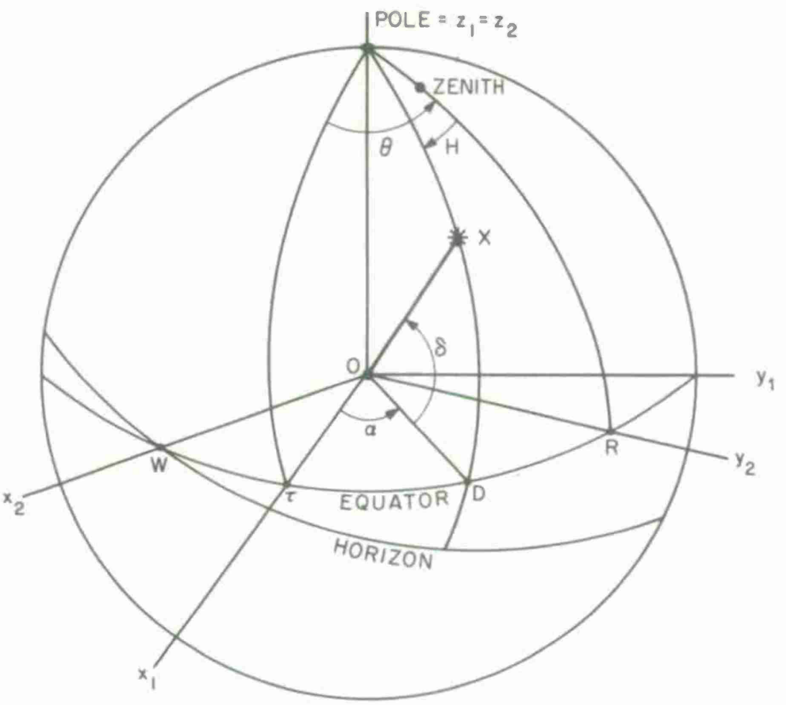

Fig. 3a. Transformation from the right ascension-declination system to the hour angle-declination system.

3-21-6101

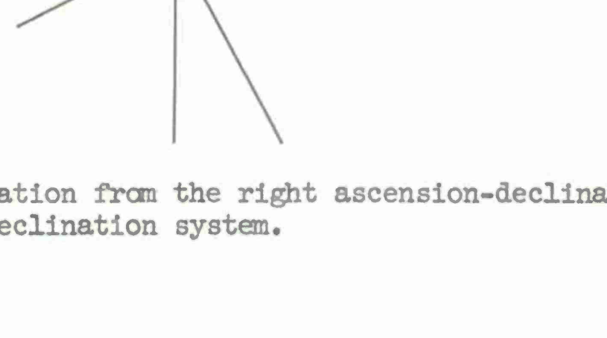

Fig. 3b. Transformation from the right ascension-declination system to the hour angle-declination system.

3-21-6102

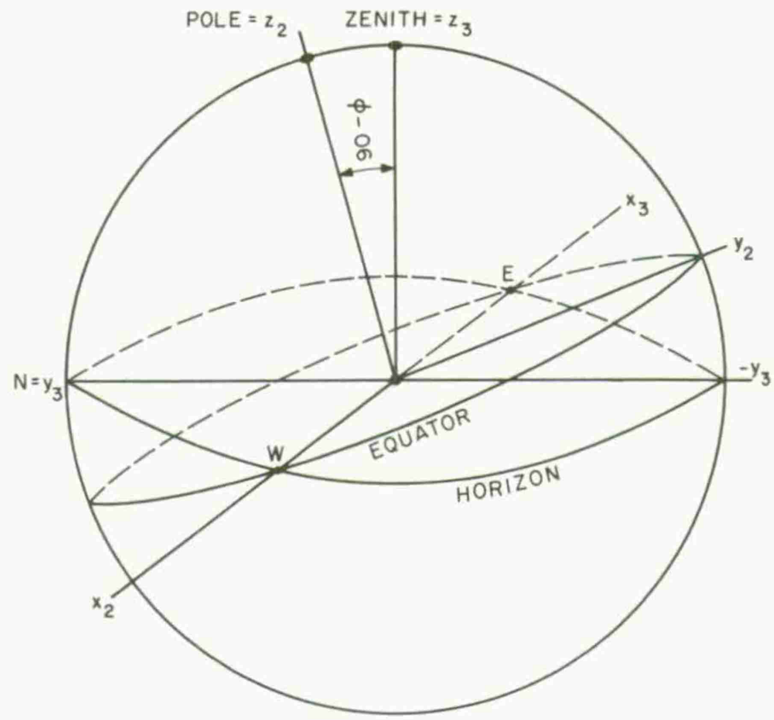

Fig. 3c. Transformation from the hour angle-declination system to the altitude-azimuth system.

3-21-6103

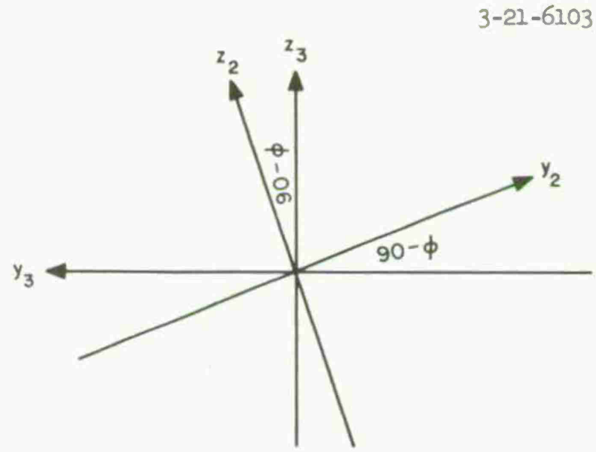

Fig. 3d. Transformation from the hour angle-declination system to the altitude-azimuth system.

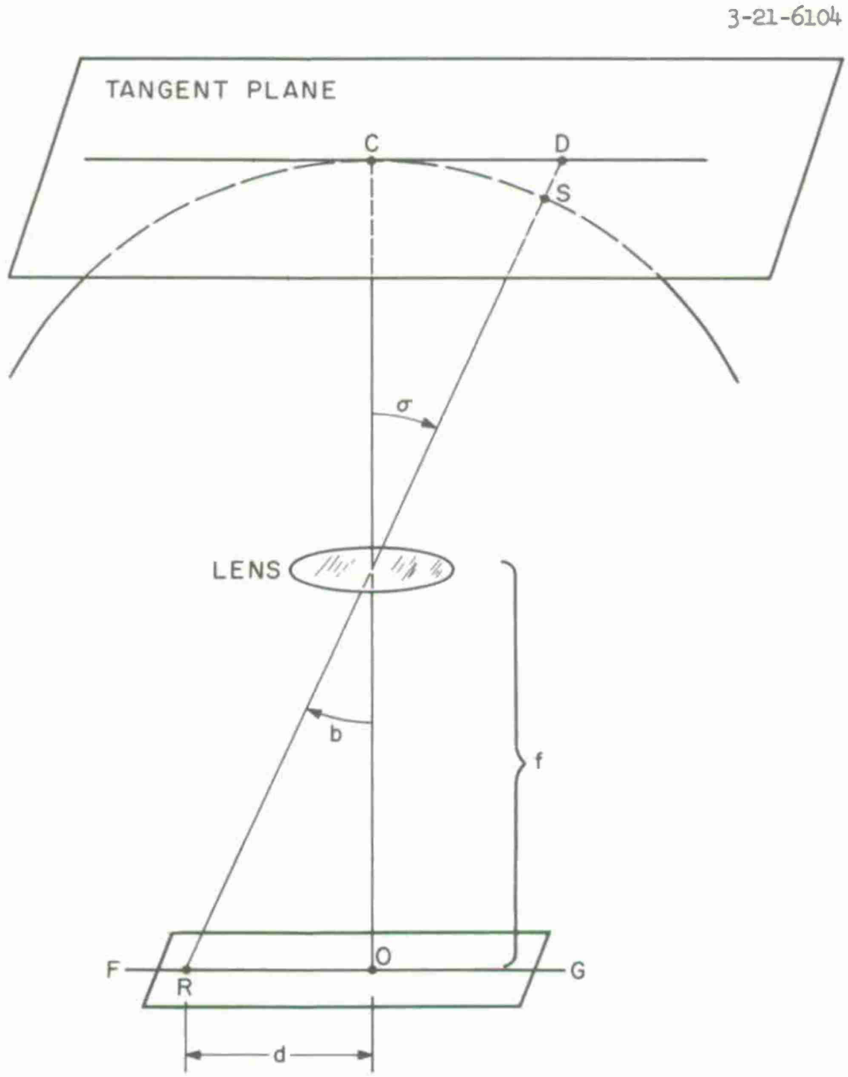

Fig. 4. Diagram of the optical system used for photographing celestial objects.

3-21-6105

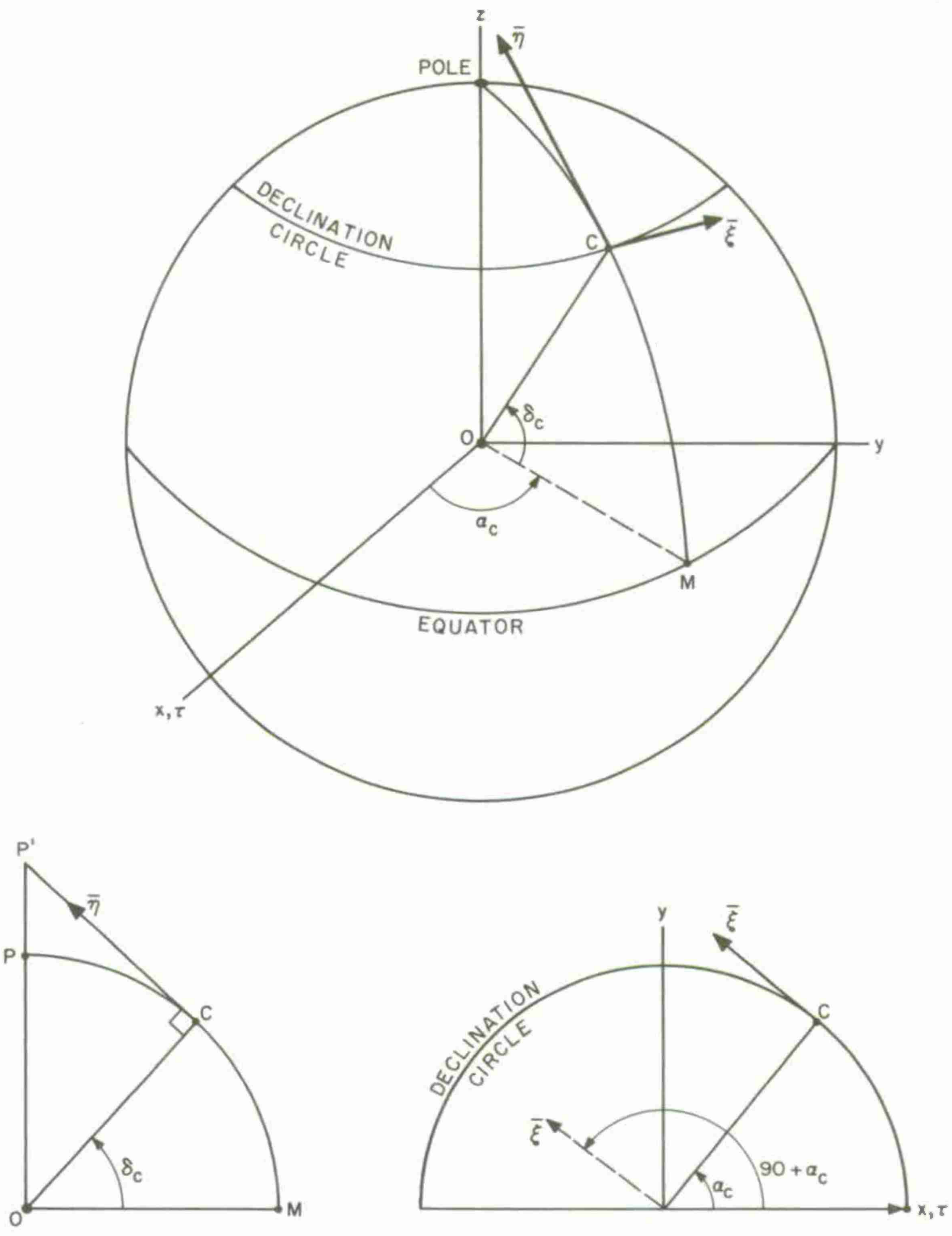

Fig. 5. Standard plate coordinate system with center at the optical center of the plate.

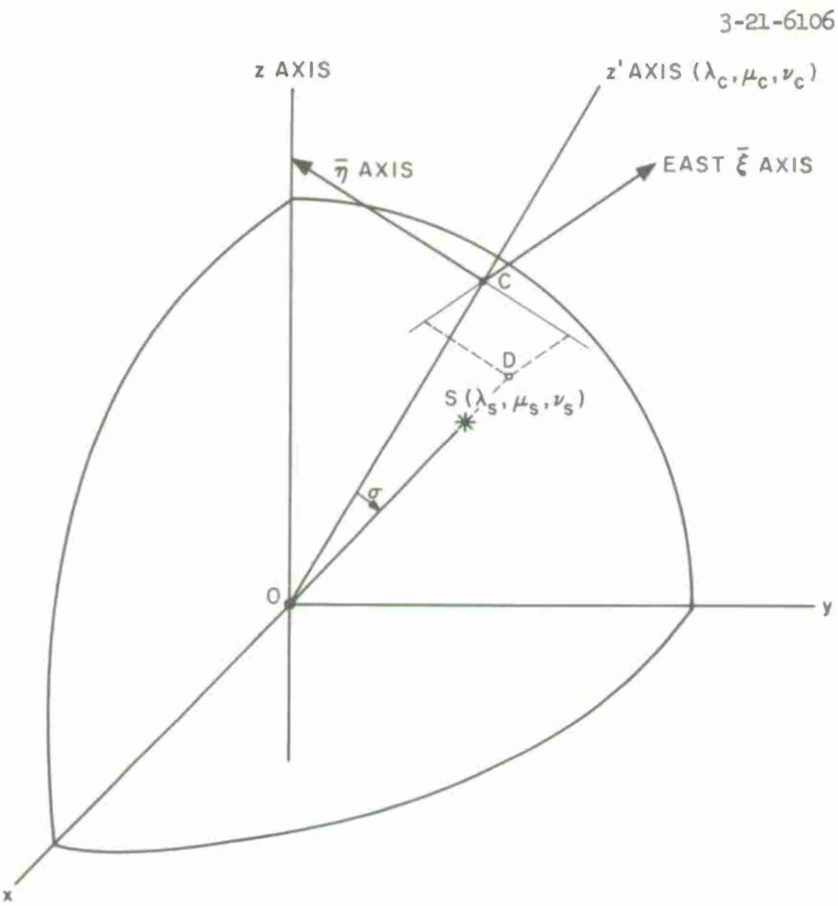

Fig. 6a. Transformation from the declination-right ascension system to the standard plate coordinate system

3-21-6107

	x	y	z
$\bar{\eta}$	λ_η	μ_η	ν_η
$\bar{\xi}$	λ_ξ	μ_ξ	0
z'	λ_c	μ_c	ν_c

Fig. 6b. Transformation from the declination-right ascension system to the standard plate coordinate system.

3-21-6108

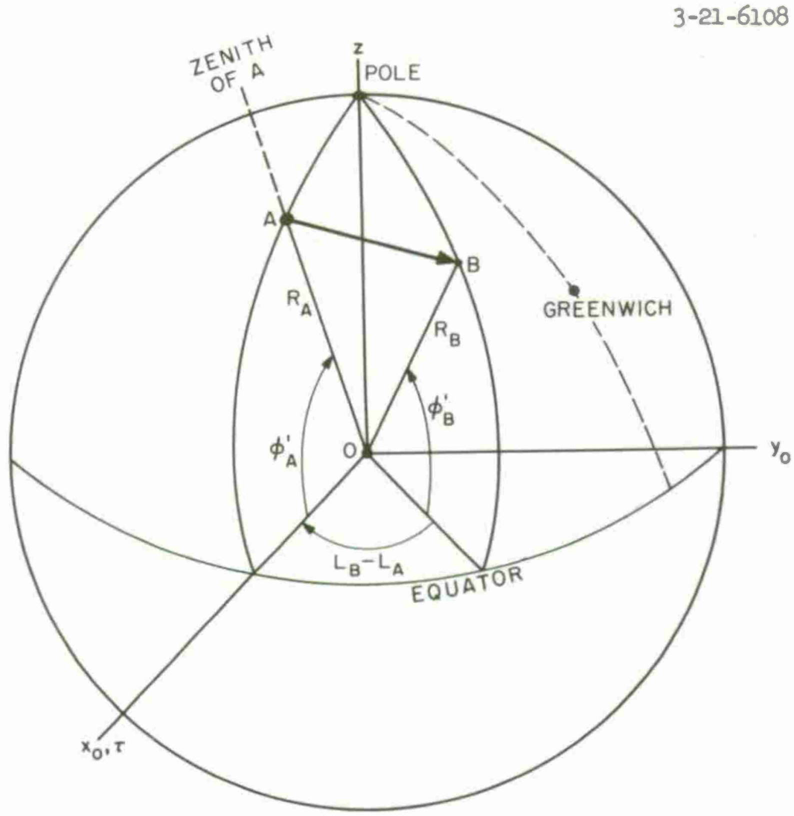

Fig. 7a. Relative coordinates of station B with respect to station A at sidereal time zero at station A.

3-21-6109

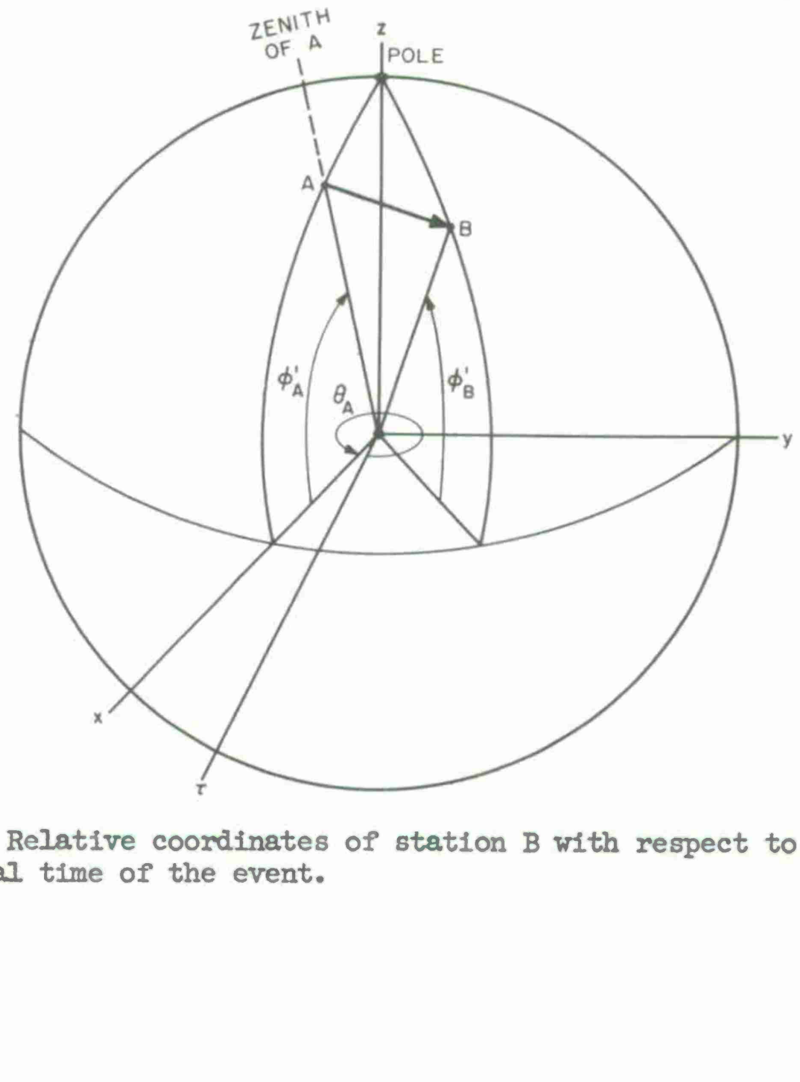

Fig. 7b. Relative coordinates of station B with respect to station A at sidereal time of the event.

3-21-6110

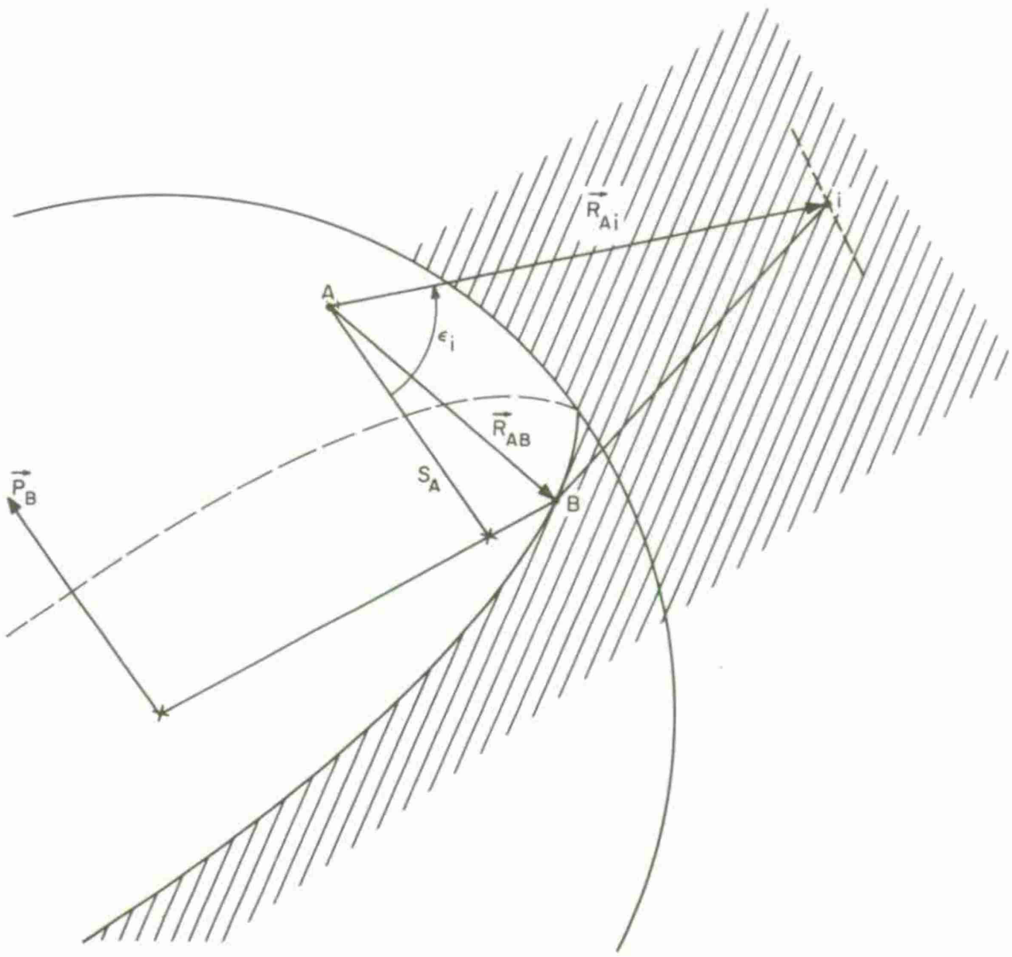

Fig. 8. Construction to show the plane through the missile trail and station B and the range vector to measured points from station A.

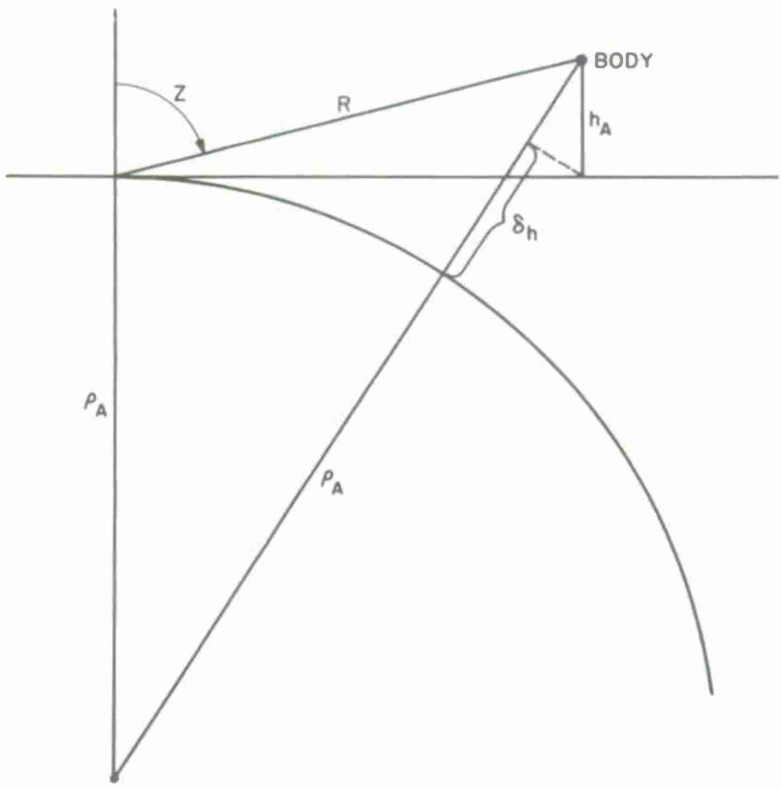

Fig. 9. Construction to show the method of computing height above mean sea level.

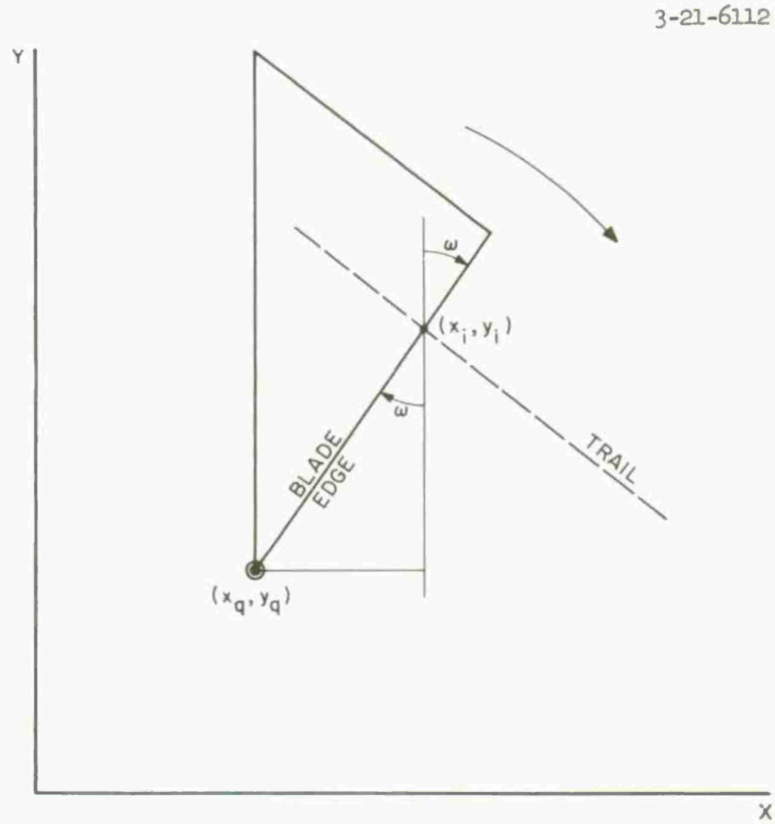

Fig. 10. The effect of the rotation of the shutter on points on the photographic plate.

3-312-7062(1)

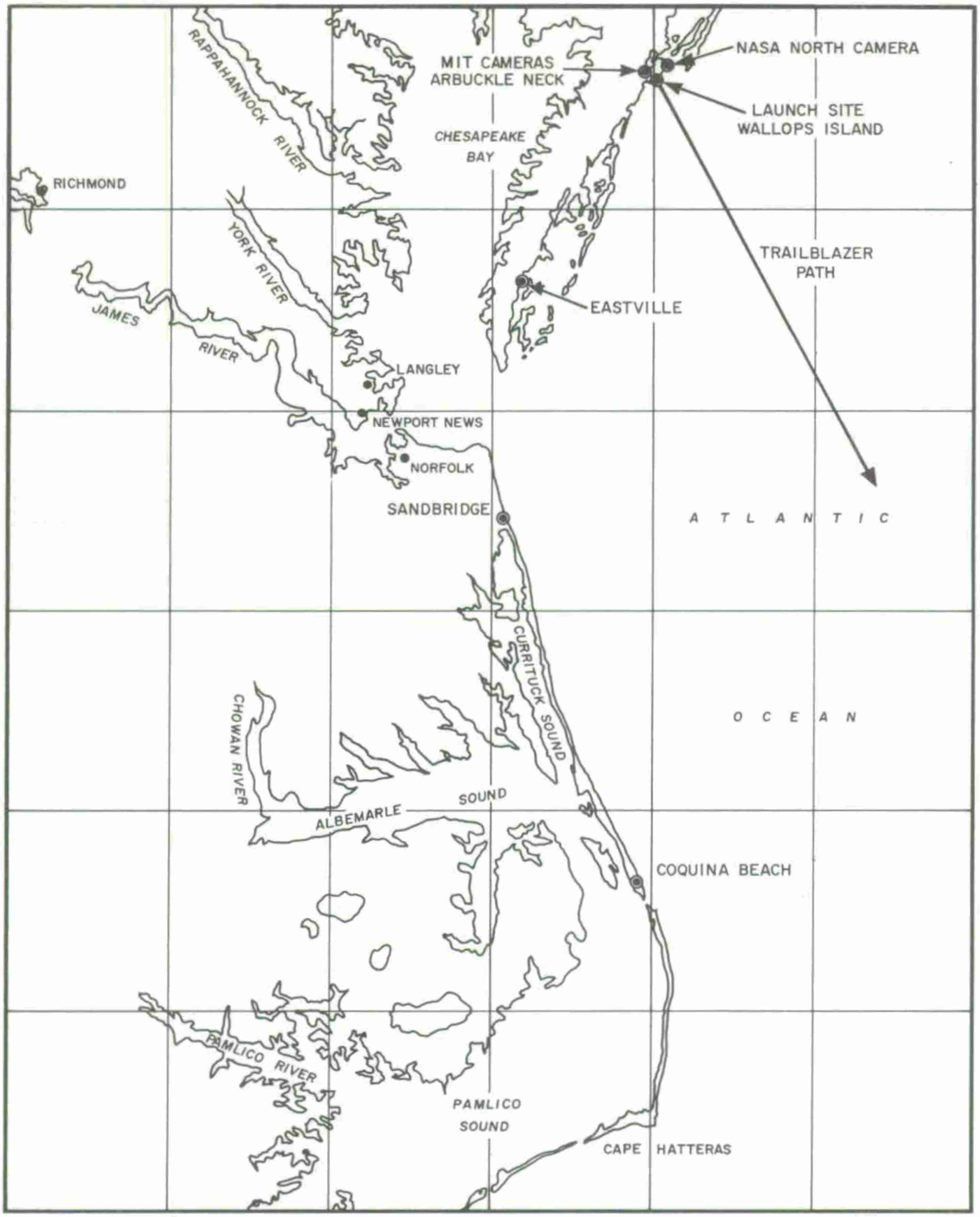

Fig. 11. Camera locations in relation to the launch site and the predicted trajectory of the missile.

P436-430

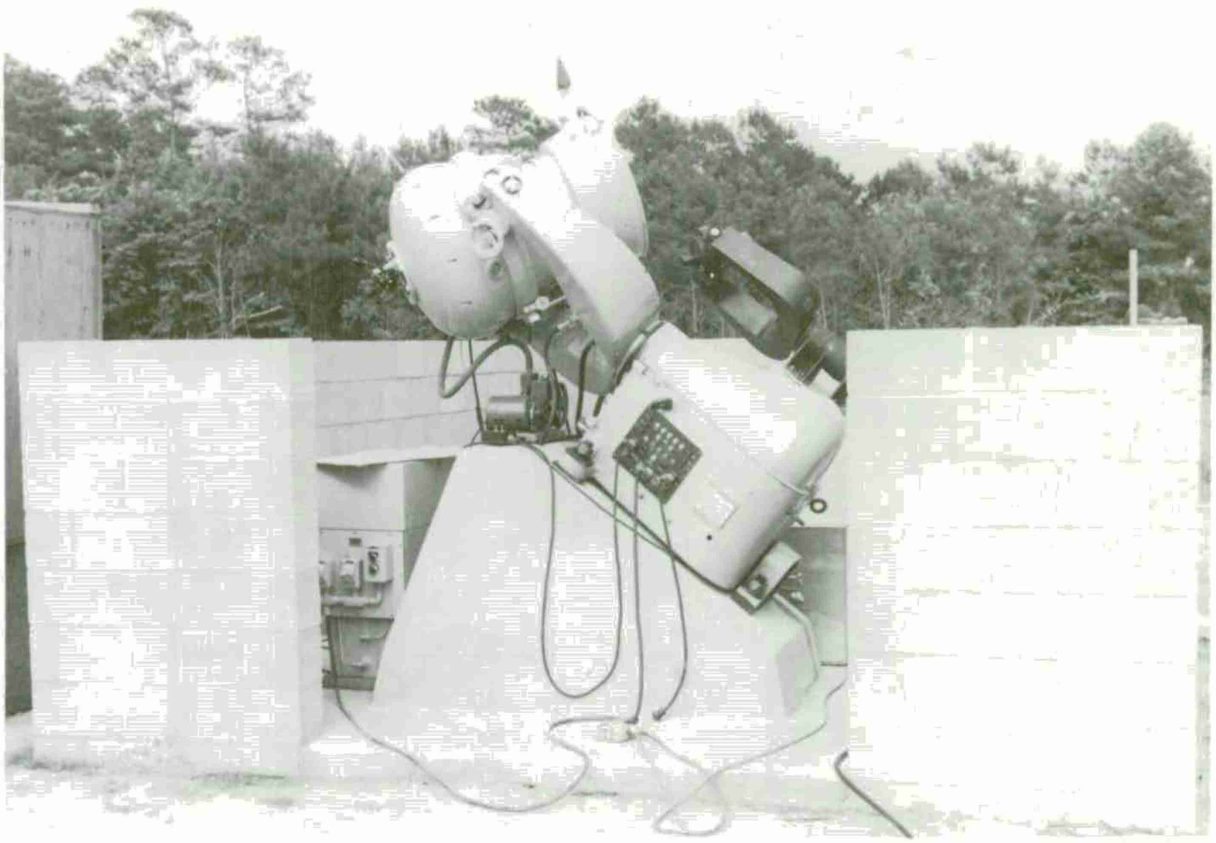

Fig. 12a. The Harvard Super-Schmidt camera at Arbuckle Neck, Virgini..
The small camera on the right is a baby Schmidt.

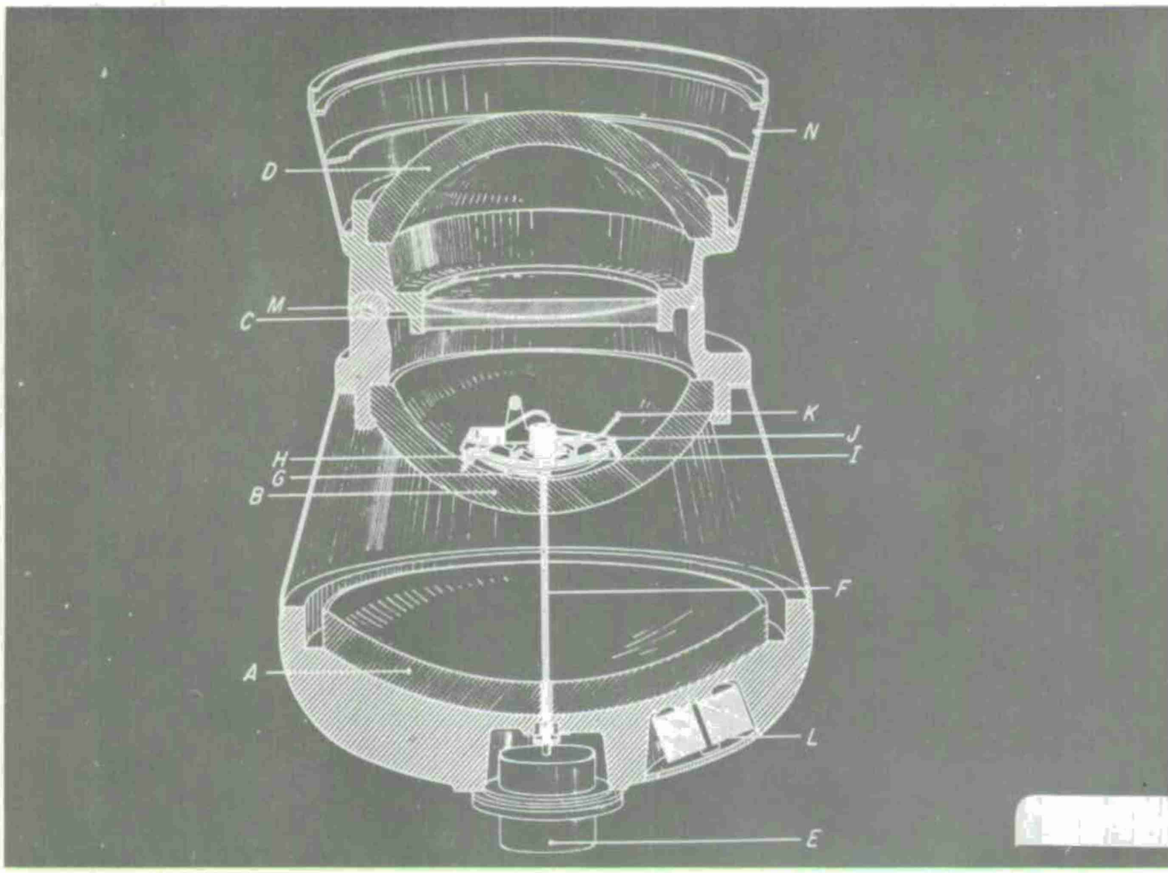

Fig. 12b. The Super-Schmidt optical system (from "Meteor Science and Engineering," Donald McKinley).

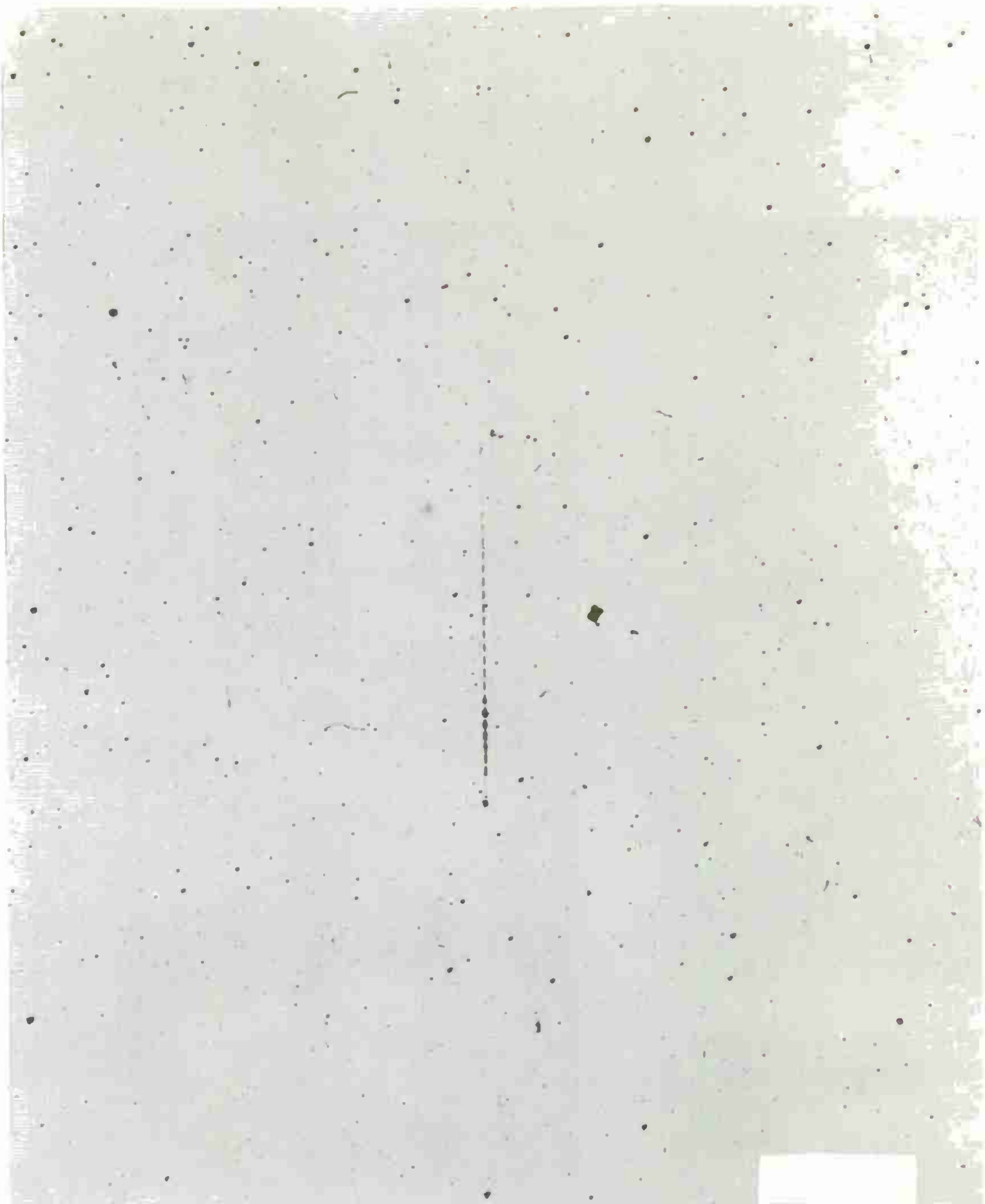

Fig. 13a. Eastville Super-Schmidt photograph of Trailblazer Ik sixth stage re-entry. Chopping rate, 15 chops per second.

Fig. 13b. Coquina Beach K 24 aerial camera photograph of Trailblazer Ic sixth stage re-entry. No chopping shutter.

C21-265

Fig. 13c. Eastville WILD BC 4 photograph of Trailblazer Ih sixth stage re-entry. No chopping shutter

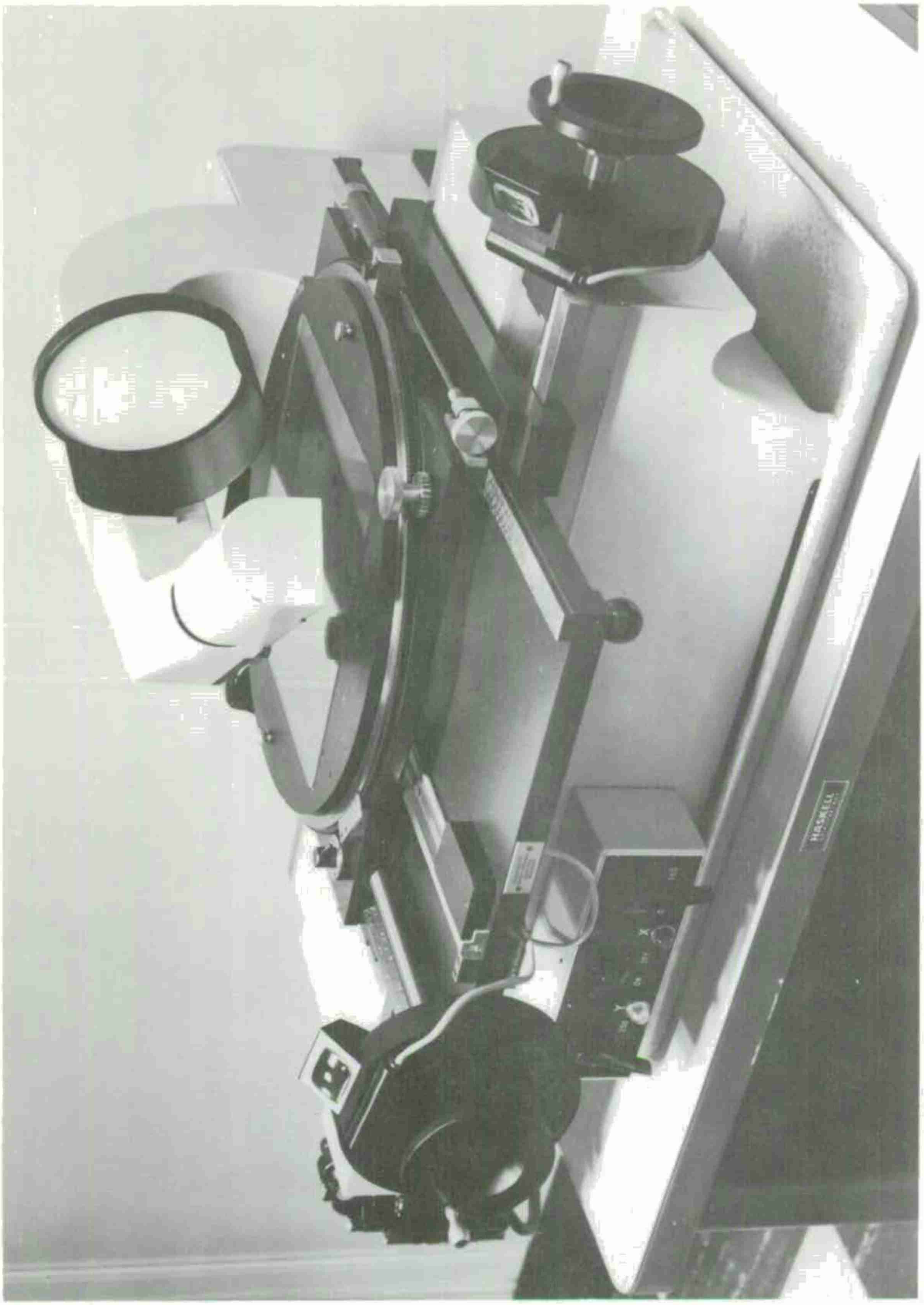

Fig. 14 A 2 screw rotatable horizontal measuring engine of a type suitable for reading ballistic plates.

3-21-6113

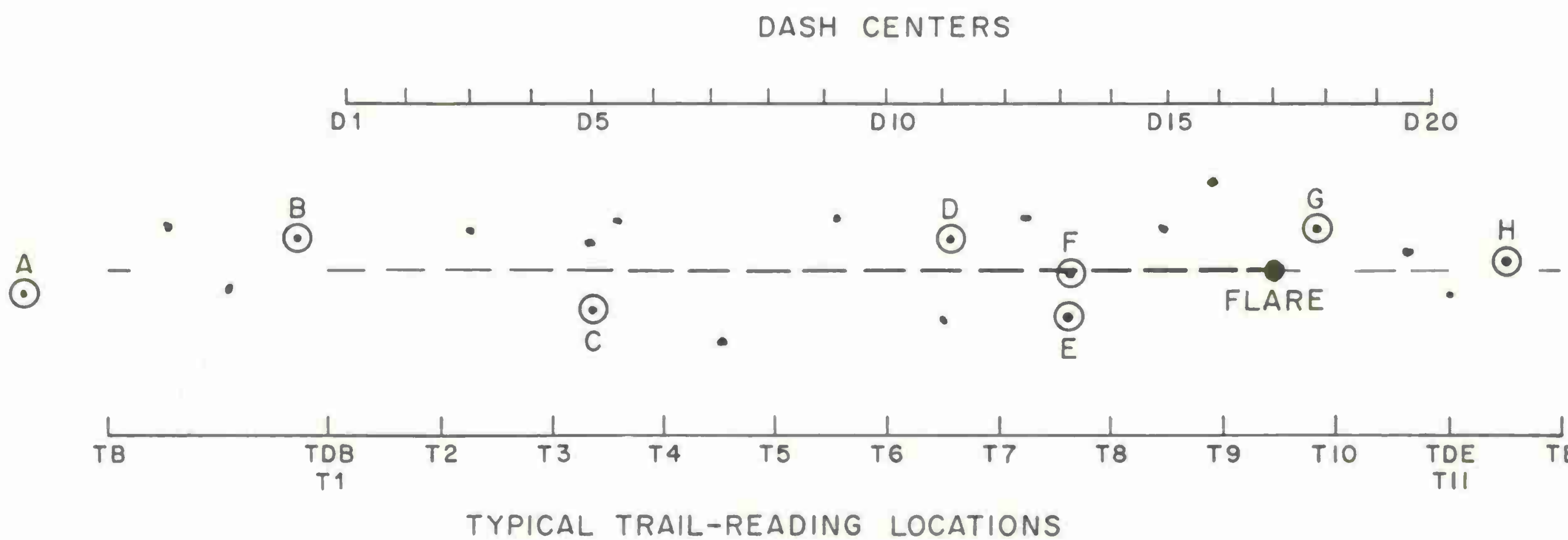

Fig. 15. Diagrammatic sketch of measurements to be made on a photographic plate.

3-21-6114

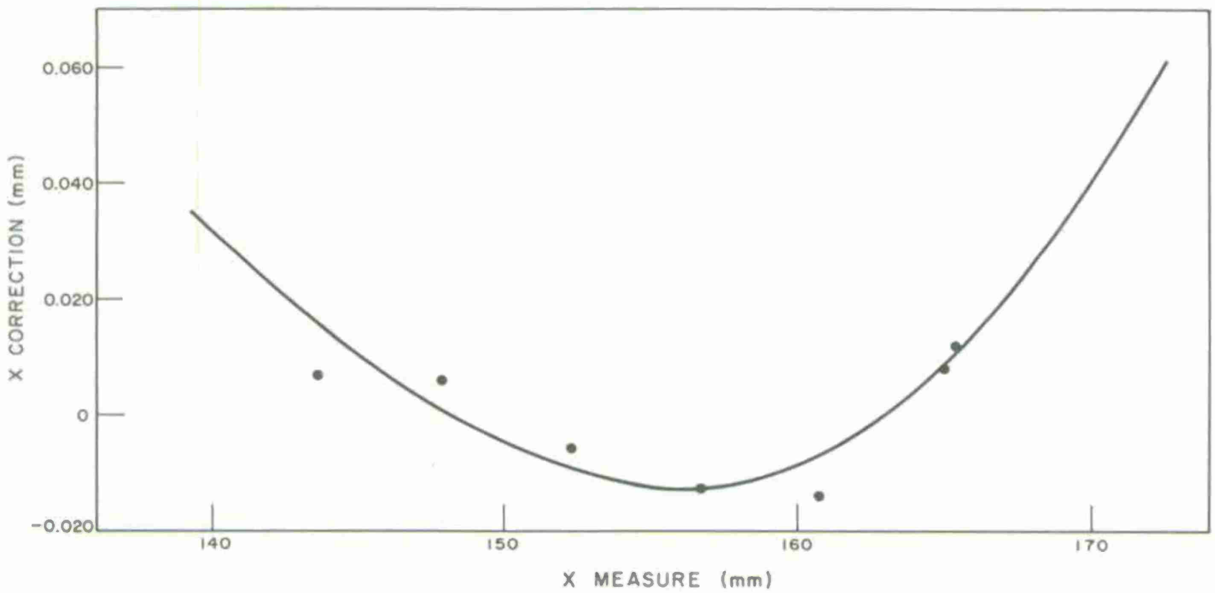

Fig. 16a. Sample curves showing residuals in X measurements.

3-21-6115

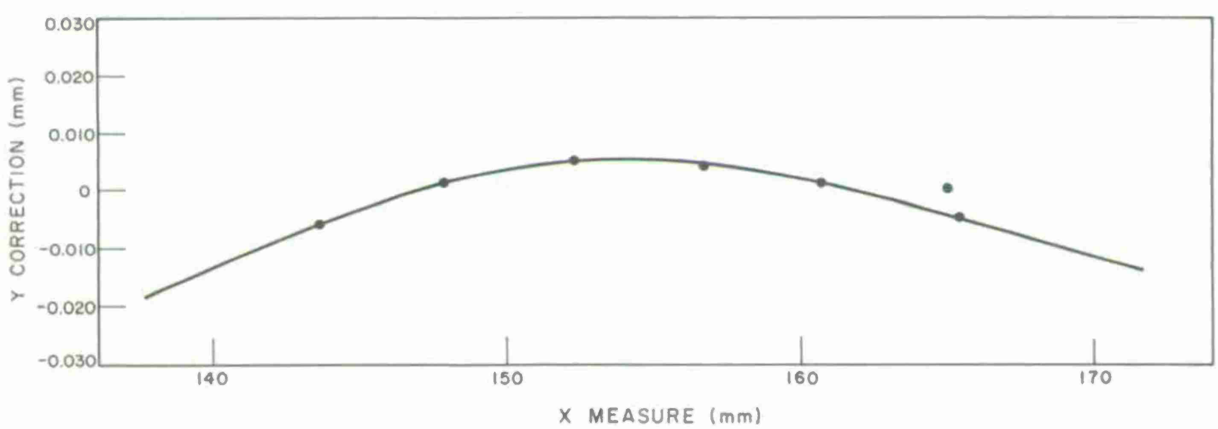

Fig. 16b. Sample curves showing residuals in Y measurements

-21-6386

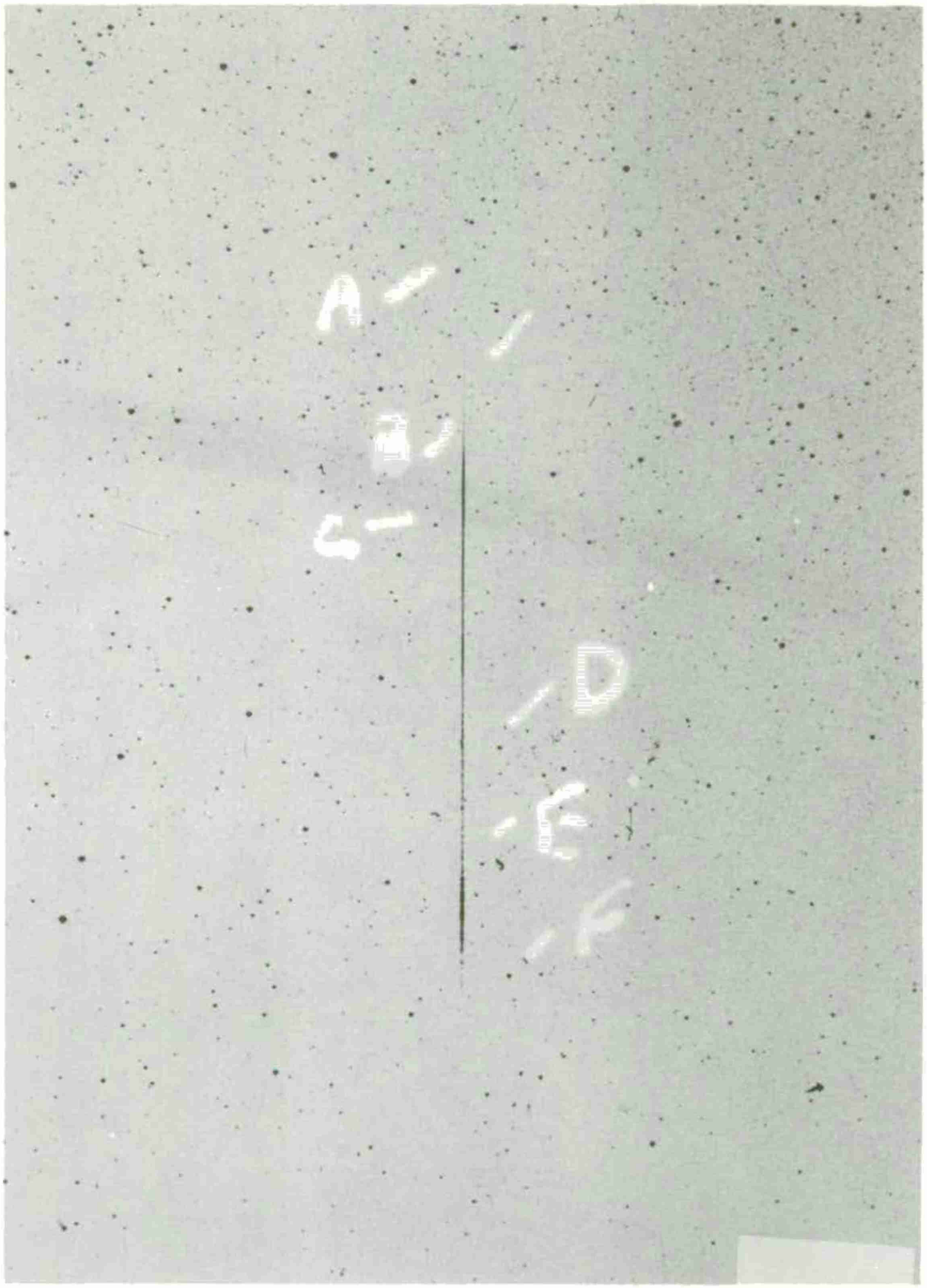

Fig. 17a. Arbuckle Neck Super-Schmidt photograph of Trailblazer Ik sixth stage re-entry. Chopping rate, 20 chops per second.

C33-851A

Fig. 17b. Eastville Super-Schmidt photograph of Trailblazer Ik sixth stage re-entry. Chopping rate, 20 chops per second.

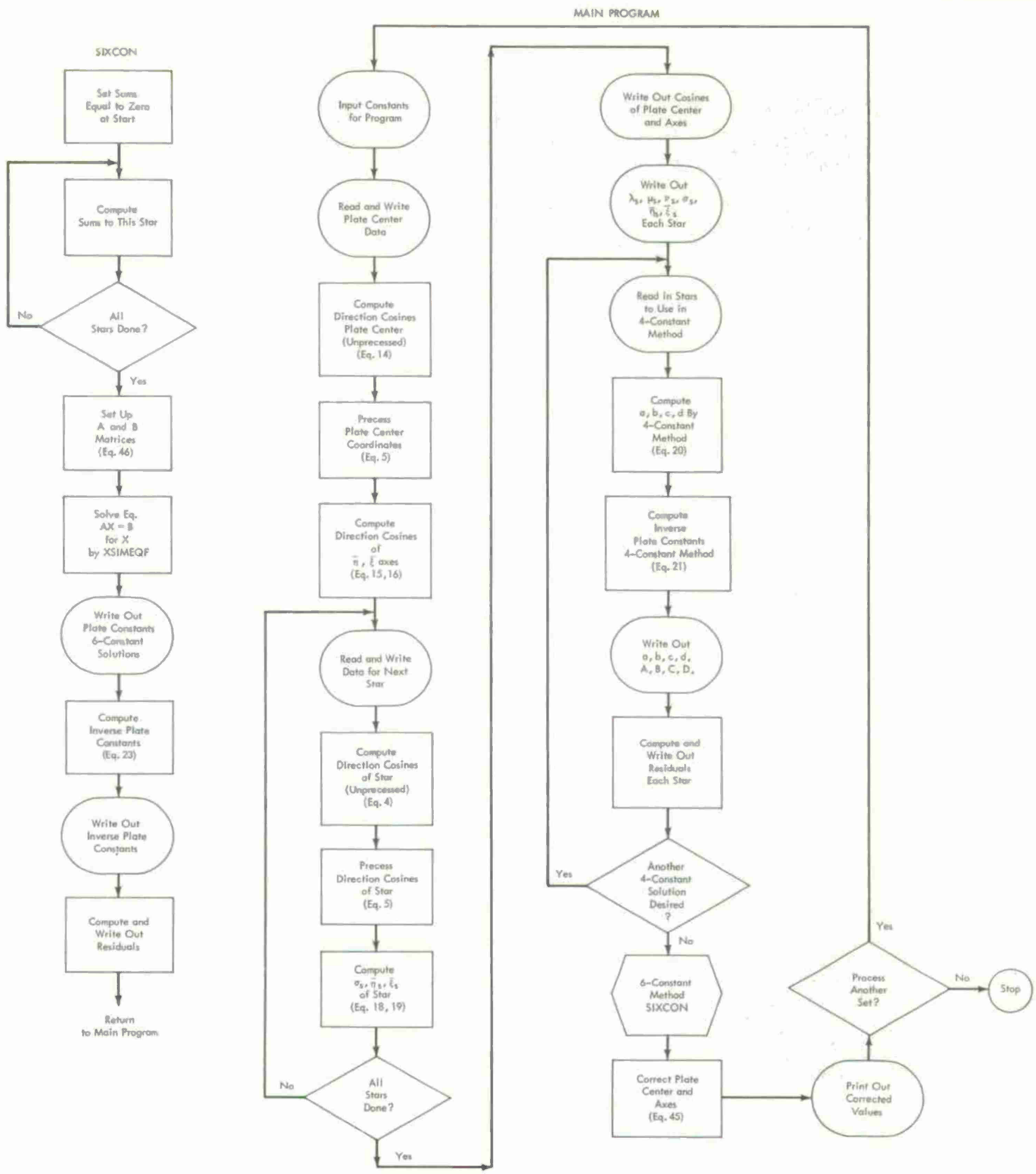

Fig. 18. Flow chart of the plate calibration program.

3-21-6117

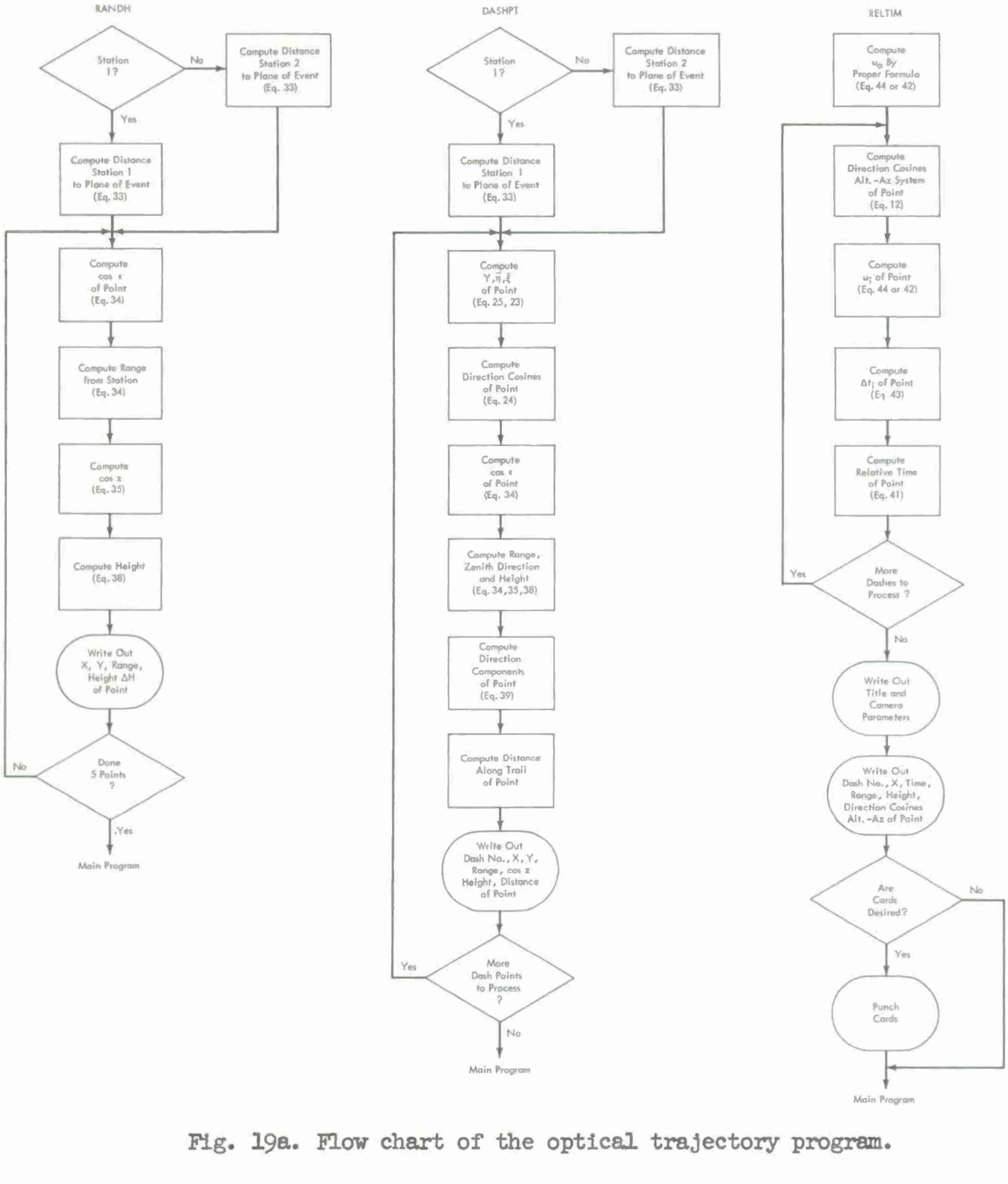

Fig. 19a. Flow chart of the optical trajectory program.

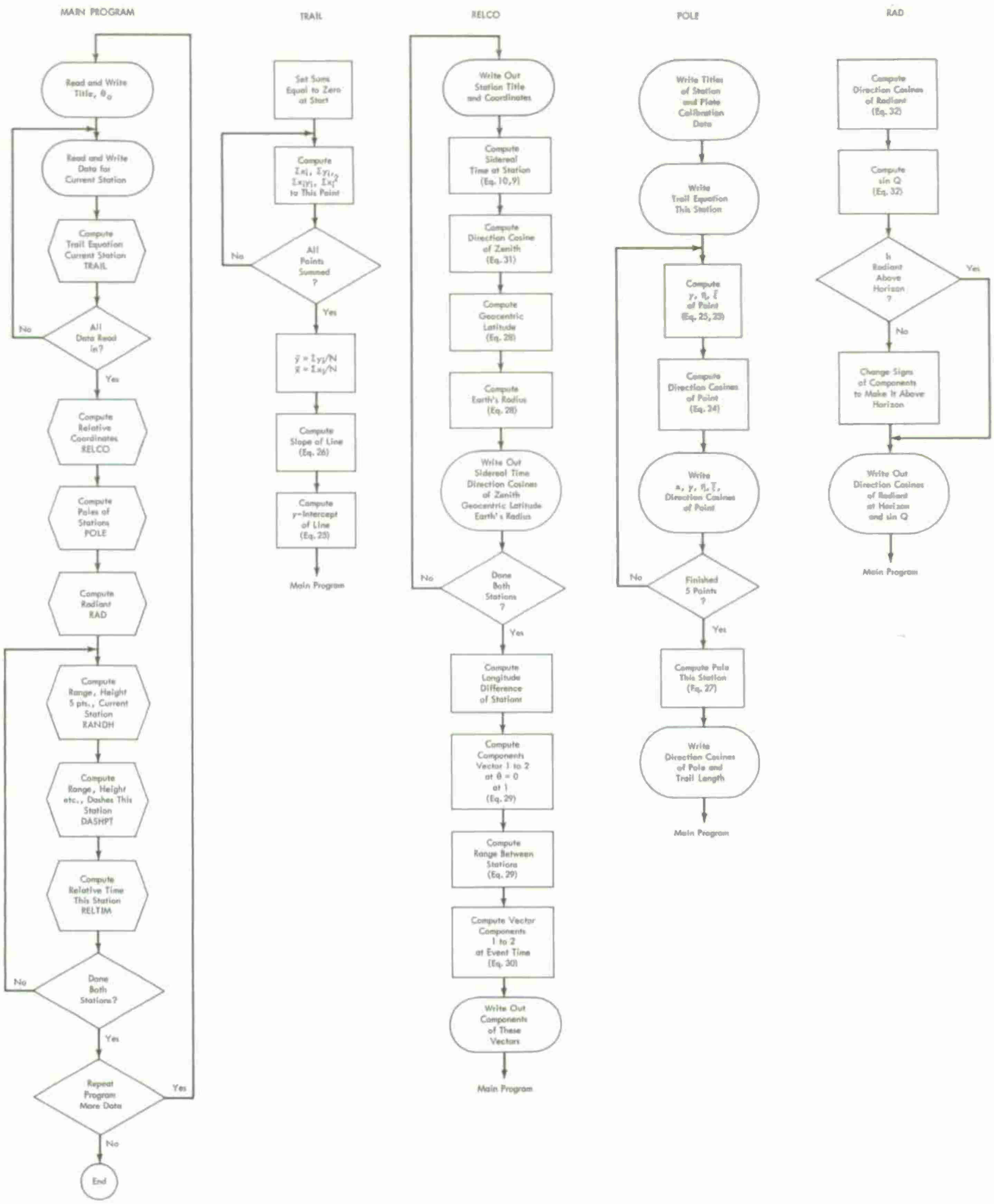

Fig. 19b. Flow chart of the optical trajectory program.

APPENDIX A

The following charts and catalogues may be used to obtain the astronomical coordinates of the stars chosen for the plate calibration.

Charts

Bonner Durchmusterung (B D)	+90° - 23°
Cordoba Resultados (C D)	-23° - -90°
Norton's Star Atlas very preliminary identification of brightest stars	

Preliminary Catalogues

Bonner Durchmusterung (B D)	- 2° - +90°
Schurnfeld's Southern Extension of Bonner Durchmusterung (BD)	- 2° - -23°
Cordoba Durchmusterung (C D) used mainly -23° - -52°	-23° - -90°
Cape Photographic Durchmusterung (C P D) used mainly -52° to -90°	-40° - -90°

Intermediate Catalogues

Henry Draper Catalogue intermediate catalogues used with the Boss General Catalogue

Final or Precision Catalogues

AGK_2 with EBL Nord (for proper motion) (Zweiter Katalog Der Astronomischen Gesellschaft)	-90° - - 2°
Yale Zone Catalogues	+90° - +85°
(Transactions of the Astronomical Observatory of	+60° - +50°
Yale University)	+30° - -30°
Cape Zone Catalogues appearing in Cape Annals	-30° - -40°
Cape Zone Catalogue with Cape Proper Motion Catalogue or	-52° - -64°
Cape Catalogue of Faint Stars	-40° - -52°
La Plata Catalogues (or Cordoba Catalogues)	-64° - -82°
Boss General Catalogue	-90° - +90°

```
1TRAILBLAZER 1K SL PLATE
    2    6
01855-1963
 0.99965377E+00-0.24128620E-01-0.10494900E-01
 0.24128620E-01+0.99970885E+00-0.12665000E-03
 0.10494900E-01-0.12661000E-03+0.99994491E+00
01950-1963
 0.99999498E+00-0.29055300E-02-0.12631600E-02
 0.29055300E-02+0.99999578E+00-0.18400000E-05
 0.12631600E-02-0.18400000E-05+0.99999920E+00
 19.+45.+00.0000-18.-17.-00.000                  1               PC
 20.+06.+13.7540-28.-35.-13.950+173.10150+020.41100    2               A
 20.+07.+28.9680-29.-54.-35.970+178.15500+020.73800    2               B
 20.+08.+17.2300-30.-56.-52.600+182.08600+021.12550    2               C
 20.+13.+45.2300-32.-45.-45.200+189.98750+018.83850    2               D
 20.+14.+25.7800-33.-50.-22.200+194.17100+019.44950    2               E
 20.+17.+02.5700-34.-44.-34.300+198.17700+018.45050    2               F
    1
    1    6
```

1

```
TRAILBLAZER 1K SL PLATE

1855-1963
   0.99965376  -0.02412862  -0.01049490   0.02412862   0.99970885  -0.00012664   0.01049490  -0.00012661   0.99994490

1950-1963
   0.99999498  -0.00290553  -0.00126316   0.00290553   0.99999578  -0.00000183   0.00126316  -0.00000183   0.99999920

RT. ASC. CENTER= 19. H 45. M  0.     S
DEC. CENTER=-18. D-17. M -0.    S
XC=  -0.          YC=  -0.
DTHETA= -0.

 RIGHT ASCENSION        DECLINATION          X              Y
 HR   MIN     SEC       DEG   MIN   SEC
  20.   6.  13.7540     -28.  -35.  -13.950     173.101        20.411
  20.   7.  28.9680     -29.  -54.  -35.970     178.155        20.738
  20.   8.  17.2300     -30.  -56.  -52.600     182.086        21.125
  20.  13.  45.2300     -32.  -45.  -45.200     189.987        18.838
  20.  14.  25.7000     -33.  -50.  -22.200     194.171        19.449
  20.  17.   2.5700     -34.  -44.  -34.300     198.177        18.450
```

2

```
TRAILBLAZER 1K SL PLATE

RT. ASC. CENTER= 19. H 45. M  0.     S
DEC. CENTER=-18. D-17. M -0.    S

LAMC=  4.4365523E-01      MUC= -8.4117501E-01      NUC= -3.0918370E-01
LAMETA=  1.4423831E-01    MUETA= -2.7347735E-01    NUETA=  9.5100232E-01
LAMEXI=  8.8451413E-01    MUEXI=  4.6651330E-01    NUEXI=  0.

NO.      LAMDA             MU               NU               COSSIG           ETA               XI
 1    4.6232523E-01   -7.4689619E-01   -4.7791356E-01   9.8114649E-01   -1.8707982E-01   6.1658682E-02
 2    4.6044530E-01   -7.3479730E-01   -4.9805903E-01   9.7636391E-01   -2.1128492E-01   6.6038591E-02
 3    4.5814687E-01   -7.2541854E-01   -5.1368208E-01   9.7228533E-01   -2.3043131E-01   6.8724664E-02
 4    4.6610159E-01   -7.0037954E-01   -5.4057167E-01   9.6306611E-01   -2.6510877E-01   8.8817451E-02
 5    4.6244786E-01   -6.9042587E-01   -5.5628589E-01   9.5793092E-01   -2.8552229E-01   9.0767314E-02
 6    4.6528450E-01   -6.7778927E-01   -5.6930831E-01   9.5258614E-01   -3.0332339E-01   1.0009910E-01
```

3

```
TRAILBLAZER 1K SL PLATE
A= 4.9899776E 01        B= -1.9947041E 02      C= 1.3275568E 02        D= 2.3524998E 01
ACAP= 1.1634350E-03     BCAP= -4.7267051E-03   GCAP= -4.3256880E-02    DCAP= 6.5486677E-01
STARS      1       6

AEXI= 1.1634350E-03     BEXI= -4.7267051E-03   CEXI= -4.3256880E-02
AETA= -4.7267051E-03    BETA= -1.1634350E-03   CETA= 6.5486677E-01

NO      X(S)      Y(S)      DX        DY
 1    173.101    20.411  -0.00000  -0.00000
 2    178.165    20.738  -0.01006  -0.01223
 3    182.086    21.125   0.01012   0.00454
 4    189.987    18.838   0.01258  -0.01388
 5    194.171    19.449  -0.00313  -0.01154
 6    198.177    18.450  -0.00000  -0.00000
```

4

```
TRAILBLAZER 1K SL PLATE

MATRIX=    1

AX= 4.8968118E 01       BX= -1.9949473E 02     CX= 1.3276051E 02
AY= -1.9877777E 02      BY= -4.0890820E 01     CY= 2.3526559E 01

AEXI= 1.1627049E-03     BEXI= -4.7443160E-03   CEXI= -4.2743869E-02
AETA= -4.7272654E-03    BETA= -1.1645432E-03   CETA= 6.5499187E-01

NO      X(S)      Y(S)      DX        DY
 1    173.101    20.411  -0.00024   0.00567
 2    178.165    20.738  -0.01048  -0.00855
 3    182.086    21.125   0.00966   0.00618
 4    189.987    18.838   0.01003  -0.00549
 5    194.171    19.449  -0.00559  -0.00605
 6    198.177    18.450  -0.00341   0.00831
```

5

```
TRAILBLAZER 1K SL PLATE

CORRECTED COORDINATES OF CENTER

LAMC= 4.4365523E-01       MUC= -8.4117500E-01       NUC= -3.0918370E-01
LAMETA= 1.4423831E-01     MUETA= -2.7347735E-01     NUETA= 9.5100232E-01
LAMEXI= 8.8451413E-01     MUEXI= 4.6651329E-01      NUEXI= 0.
```

```
1TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS                                      TITLE
 20.+20.+43.0380
OSL PLATE ARBUCKLE NECK
 02.+56.+21.583 +37.+51.+23.2660+05.+02.+02.7830+00.0000000                    COOR A
   15                                                                          N TR A
 0.17600000E+03+0.20140000E+02                                                 XO YO A
 0.00000000E+00+0.00000000E+00+0.15000000E+01+0.10000000E-02                   TR1   A
 0.30000000E+01+0.80000000E-02+0.45000000E+01+0.20000000E-02                   TR2   A
 0.60000000E+01+0.60000000E-02+0.75000000E+01+0.20000000E-02                   TR3   A
 0.90000000E+01-0.10000000E-02+0.10500000E+02+0.00000000E+00                   TR4   A
 0.12000000E+02+0.40000000E-02+0.13500000E+02+0.20000000E-02                   TR5   A
 0.15000000E+02+0.00000000E+00+0.16500000E+02-0.10000000E-02                   TR6   A
 0.18000000E+02-0.30000000E-02+0.19500000E+02+0.00000000E+00                   TR7   A
 0.21000000E+02-0.40000000E-02                                                 TR8   A
 0.11627049E-02-0.47443160E-02-0.42743869E-01                                  EXIS  A
 0.47272654E-02-0.11645432E-02+0.65499187E+00                                  ETAS  A
 0.44365523E+00+0.88451413E+00+0.14423830E+00                                  LAMB  A
 0.84117500E+00+0.46651330E+00-0.27347735E+00                                  MU    A
 0.30918370E+00+0.95100233E+00  .                                              NU    A
 175.86100+187.74500+194.74700+197.73400+198.64400                             5 XS  A
   89
    0     1+175.86100     1      4+175.94600     2     4+176.24500             DASH1 A
    3     1+176.53400     4      2+176.79600     5     1+177.05700             DASH2 A
    6     2+177.33200     7      4+177.59900     8     4+177.90000             DASH3 A
    9     3+178.16400    10      4+178.44300    11     4+178.72800             DASH4 A

                              DATA MISSING

   87     1+198.56700     0      1+198.64400                                   DASH30 A
 0.13320500E+03+0.23520000E+02+0.13320500E+03+0.23500000E+02
 0.40740000E+04+0.10000000E+00+0.20000000E+01                                  CAM2    A
    0-    1
OSS PLATE EASTVILLE                                                            TITLE B
 02.+56.+21.583 +37.+20.+46.4300+05.+03.+36.7650+00.0000000                    COOR  B
   15                                                                          N TR  B
 0.18000000E+03+0.44885000E+02                                                 XO YO B
 0.00000000E+00+0.20000000E-01+0.15000000E+01+0.40000000E-02                   TR1   B
 0.30000000E+01-0.10000000E-02+0.45000000E+01+0.10000000E-02                   TR2   B
 0.60000000E+01-0.50000000E-02+0.75000000E+01-0.80000000E-02                   TR3   B
 0.90000000E+01-0.90000000E-02+0.10500000E+02-0.10000000E-02                   TR4   B
 0.12000000E+02-0.50000000E-02+0.13500000E+02-0.13000000E-01                   TR5   B
 0.15000000E+02-0.70000000E-02+0.16500000E+02-0.60000000E-02                   TR6   B
 0.18000000E+02-0.30000000E-02+0.19500000E+02+0.50000000E-02                   TR7   B
 0.20100000E+02-0.10000000E-02                                                 TR8   B
 0.25734744E-02-0.41302408E-02-0.28662772 +00                                  EXIS  B
 0.41605029E-02-0.25337586E-02+0.54358602 +00                                  ETAS  B
 0.75910489E+00+0.65095623E+00+0.39516512 -02                                  LAMB  B
 0.65094741E+00+0.75911517E+00-0.33886189 -02                                  MU    B
 0.52056018E-02+0.99998645E+00                                                 NU    B
 176.84700+186.71700+199.26900+202.44800+202.52500                             5 XS  B
   51
    0     1+176.84700     1      1+186.71700     2     1+186.97500             DASH1 B
    3     1+187.36800     4      1+187.66000     5     1+187.96200             DASH2 B

                              DATA MISSING

   52     1+202.22000    53      1+202.44800     0     1+202.52500             DASH17 B
 0.12665600E+03+0.10333000E+02+0.12665600E+03+0.10333000E+02                   CAM1    B
 0.40740000E+04+0.10000000E+00+0.20000000E+01                                  CAM2    B
    0+    1
```

```
TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS

THETA= 20.H 20.M 43.03806

SL PLATE ARBUCKLE NECK
TIME(1)= 21.H 56.M 21.58309
PHI(1)= 37.D 51.M 23.2660S
LONG(1)= 5.H 2.M 2.7830S
ELEV(1)= 0.        KFT
 0.17600000E 03 0.20140000E 02
 0.            0.            0.15000000E 01 0.99999999E-03
 0.38000000E 01 0.79999999E-02 0.45000000E 01 0.20000000E-02
 0.59999999E 01 0.59999999E-02 0.74999999E 01 0.20000000E-02
 0.89999999E 01-0.99999999E-03 0.10500000E 02 0.
 0.12000000E 02 0.39999999E-02 0.13500000E 02 0.20000000E-02
 0.15000000E 02 0.             0.16500000E 02-0.99999999E-03
 0.18000000E 02-0.30000000E-02 0.19499999E 02 0.
 0.20999999E 02-0.39999999E-02
 0.11627048E-02-0.47443160E-02-0.42743868E-01
-0.47272654E-02-0.11645432E-02 0.65499187E 00
 0.44365522E-00 0.88451412E 00 0.14423829E-00
-0.84117500E 00 0.46651330E-00-0.27347735E-00
-0.38918369E-00 0.95100233E 00
 175.86100 187.74500 194.74700 197.73400 198.64400
    0    1 175.86100    1    4 175.94600    2    4 176.24500
    3    1 176.53400    4    2 176.79600    5    1 177.05700


   87    1 198.56700    0    1 198.64400
 0.13320500E 03 0.23520000E 02 0.13320500E 03 0.23500000E 02
 0.40740000E 04 0.09999999E-00 0.20000000E 01
   -1

SS PLATE EASTVILLE
TIME(2)= 21.H 56.M 21.5830S
PHI(2)= 37.D 20.M 46.4300S
LONG(2)= 5.H 3.M 30.7650S
ELEV(2)= 0.        KFT
 0.18000000E 03 0.44884999E 02
 0.            0.20000000E-01 0.15000000E 01 0.39999999E-02
 0.59999999E 01-0.49999999E-02 0.74999999E 01-0.79999999E-02
 0.89999999E 01-0.89999999E-02 0.10500000E 02-0.99999999E-03
 0.30000000E 01-0.99999999E-03 0.45000000E 01 0.99999999E-03
 0.12000000E 02-0.49999999E-02 0.13500000E 02-0.13000000E-01
 0.15000000E 02-0.70000000E-02 0.16500000E 02-0.59999999E-02
 0.18000000E 02-0.30000000E-02 0.19499999E 02 0.49999999E-02
 0.20100000E 02-0.99999999E-03
 0.25734743E-02-0.41302408E-02-0.28662772E-00
-0.41605029E-02-0.25337586E-02 0.54358602E 00
 0.75910489E 00 0.65095623E 00 0.39516512E-02
-0.65094741E 00 0.75911517E 00-0.33886189E-02
-0.52056017E-02 0.99998645E 00
 176.84700 186.71700 199.26900 202.44800 202.52500
    0    1 176.84700    1    1 186.71700    2    1 186.97500
    3    1 187.36800    4    1 187.66000    5    1 187.96200
```

```
  52 .   1 202.22000   53    1 202.44800    0    1 202.52500
 0.12665600E 03 0.10332999E 02 0.12665600E 03 0.10332999E 02
 0.40740000E 04 0.09999999E-00 0.20000000E 01
    1
```

```
TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS

SL PLATE ARBUCKLE NECK
TIME(1)= 21.H 56.M 21.5830S
PHI(1)= 37.D 51.M 23.2660S
LONG(1)=  5.H  2.M  2.7830S
ELEV(1)=  0.          KFT
THETA(1)=  6.5965866E 02 DEG
ELZ(1)= -7.4353057E-01       EMZ(1)= -2.6561727E-01       ENZ(1)=  6.1368540E-01
PHIPRIME(1)=  3.7469356E 01 DEG
RHO(1)=  2.0900031E 04 KFT

SS PLATE EASTVILLE
TIME(2)= 21.H 56.M 21.5830S
PHI(2)= 37.D 20.M 46.4300S
LONG(2)=  5.H  3.M 36.7650S
ELEV(2)=  0.          KFT
THETA(2)=  5.5926700E 02 DEG
ELZ(2)= -7.5045788E-01       EMZ(2)= -2.6232229E-01       ENZ(2)=  6.0663003E-01
RHIPRIME(2)=  3.7160012E 01 DEG
RHO(2)=  2.0900636E 04 KFT

EXIAB0=  1.1297827E 02 KFT    ETAAB0= -1.1384110E 02 KFT
EXIAB= -1.4469111E 02 KFT   ETAAB=  6.9198042E 01 KFT    ZETAAB= -1.4720337E 02 KFT
ERROR=  0.              KFT
```

```
TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS

SL PLATE ARBUCKLE NECK

CAX=  1.1627049E-03   CBX= -4.7443100E-03   CCX= -4.2743868E-02
CAY= -4.7272654E-03   CBY= -1.1645432E-03   CCY=  6.5499187E-01
ELC=  4.4365522E-01   ELEXI=  8.8451412E-01   ELETA=  1.4423830E-01
EMC= -8.4117500E-01   EMEXI=  4.6651329E-01   EMETA= -2.7347735E-01
ENC= -3.0918370E-01   ENETA=  9.5100232E-01

Y= -2.7142855E-04 X  +  2.0191687E 01
```

X	Y	EL	EM	EN	ERROR
175.86100	20.14395	4.6320682E-01	-7.3946494E-01	-4.8849886E-01	-1.1175871E-07
187.74500	20.14073	4.6119066E-01	-7.0880274E-01	-5.3376189E-01	-8.1956387E-08
194.74700	20.13883	4.5940169E-01	-6.9016676E-01	-5.5912415E-01	-1.1175871E-07
197.73400	20.13802	4.5851187E-01	-6.8211490E-01	-5.6963675E-01	-9.6857548E-08
198.64400	20.13777	4.5822631E-01	-6.7965131E-01	-5.7280245E-01	-1.0430813E-07

```
ELP=  8.8591998E-01      EMP=  4.0138974E-01      ENP=  2.3244799E-01
SINL=  1.0334859E-01
TRAIL LENGTH=  5.9320299E 00 DEG

ERROR=  0.               1.8626451E-09    -3.7252903E-09
```

TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS

SS PLATE EASTVILLE

CAX= 2.5734744E-03 CBX= -4.1302407E-03 CCX= -2.8662772E-01
CAY= -4.1605029E-03 CBY= -2.5337586E-03 CCY= 5.4358602E-01
ELC= 7.5910488E-01 ELEXI= 6.5095623E-01 ELETA= 3.9516512E-03
EMC= -6.5094740E-01 EMEXI= 7.5911517E-01 EMETA= -3.3886189E-03
ENC= -5.2056017E-03 ENETA= 9.9998645E-01

Y= -4.6962026E-04 X + 4.4972500E 01

X	Y	EL	EM	EN	ERROR
176.04700	44.88945	7.1411376E-01	-6.3367878E-01	-2.9747698E-01	-1.6391277E-07
186.71700	44.88481	7.2106860E-01	-6.0775406E-01	-3.3270845E-01	-1.6391277E-07
199.26900	44.87892	7.2770266E-01	-5.7410778E-01	-3.7529845E-01	-1.5646219E-07
202.44800	44.87743	7.2900918E-01	-5.6551241E-01	-3.8566981E-01	-1.3411045E-07
202.52500	44.87739	7.2903903E-01	-5.6530397E-01	-3.8591888E-01	-1.4156103E-07

ELP= 6.7834609E-01 EMP= 5.2145812E-01 ENP= 5.1761764E-01
SINE= 1.1260283E-01
TRAIL LENGTH= 6.4653790E 00 DEG

ERROR= 0. 9.3132257E-09 9.3132257E-09

TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS

RADIANT

ELR= 2.3644279E-01 EMR= -8.2194099E-01 ENR= 5.1817740E-01
SINQ= 3.6606961E-01
ERROR= -7.4505806E-09

TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS

SL PLATE ARBUCKLE NECK

X	Y	R	H	DH
175.06100	20.14395	4.2641687E 02	-1.8743069E 02	3.5095215E 00
187.74500	20.14073	4.1514006E 02	-1.9698637E 02	3.1945801E 00
194.74700	20.13883	4.0945571E 02	-2.0226310E 02	3.0319824E 00
197.73400	20.13802	4.0722896E 02	-2.0444007E 02	2.9672852E 00
198.64400	20.13777	4.0657312E 02	-2.0509491E 02	2.9479980E 00

TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS

3L PEATE ARBUCKLE NECK

DASH	X	Y	R	COSZ	H	D
0	175.86100	20.14395	4.2641687E 02	-4.4777826E-01	-1.8743069E 02	0.
1	175.94600	20.14393	4.2632837E 02	-4.4803252E-01	-1.8750214E 02	1.9072623E-01
2	176.24500	20.14385	4.2601799E 02	-4.4892594E-01	-1.8775200E 02	8.6076735E-01
3	176.53400	20.14377	4.2571939E 02	-4.4978818E-01	-1.8799331E 02	1.5071033E 00
4	176.79600	20.14370	4.2544985E 02	-4.5056872E-01	-1.8821173E 02	2.0919503E 00
5	177.05700	20.14363	4.2518246E 02	-4.5134522E-01	-1.8842848E 02	2.6735273E 00
6	177.33200	20.14355	4.2490190E 02	-4.5216220E-01	-1.8865680E 02	3.2851821E 00
7	177.59900	20.14348	4.2463066E 02	-4.5295429E-01	-1.8887832E 02	3.8779456E 00
85	198.18700	20.13789	4.0690119E 02	-5.1050280E-01	-2.0476643E 02	4.6555334E 01
86	198.42100	20.13783	4.0674006E 02	-5.1108939E-01	-2.0492741E 02	4.6988972E 01
87	198.56700	20.13779	4.0662822E 02	-5.1149741E-01	-2.0503933E 02	4.7290608E 01
0	198.64400	20.13777	4.0657312E 02	-5.1169864E-01	-2.0509491E 02	4.7439383E 01

NO CHECKS AT END OF DASHPT

DIVIDE CHECK AT DY/DX

TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS

3L PEATE ARBUCKLE NECK

XQ= 1.3320500E 02 YQ= 2.3520000E 01 XC= 1.3320500E 02 YC= 2.3500000E 01
F2= 4.0740000E 03 P= 1.0000000E-01 OCCULT= 2.0000000E 00

DASH	X	TIME	RANGE	HEIGHT	ALPHA	BETA	GAMMA
0	175.861	0.	4.2641687E 02	-1.8743069E 02	8.5219424E-01	-2.7066493E-01	-4.4777826E-01
1	175.946	5.0002480E-02	4.2632837E 02	-1.8750214E 02	8.5190928E-01	-2.7088936E-01	-4.4803251E-01
2	176.245	1.0001113E-01	4.2601799E 02	-1.8775200E 02	8.5126750E-01	-2.7167822E-01	-4.4892594E-01
3	176.534	1.5001937E-01	4.2571939E 02	-1.8799331E 02	8.5056869E-01	-2.7243975E-01	-4.4978817E-01
4	176.796	2.0002676E-01	4.2544985E 02	-1.8821173E 02	8.4993417E-01	-2.7312933E-01	-4.5056871E-01
5	177.057	2.5003403E-01	4.2518246E 02	-1.8842840E 02	8.4930116E-01	-2.7381552E-01	-4.5134522E-01
6	177.332	3.0004159E-01	4.2490190E 02	-1.8865680E 02	8.4863318E-01	-2.7453768E-01	-4.5216219E-01
7	177.599	3.5004885E-01	4.2463066E 02	-1.8887832E 02	8.4798372E-01	-2.7523803E-01	-4.5295428E-01
85	198.187	4.2504293E 00	4.0690119E 02	-2.0476643E 02	7.9540558E-01	-3.2667529E-01	-5.1050280E-01
86	198.411	4.3004321E 00	4.0674006E 02	-2.0492741E 02	7.9481066E-01	-3.2720565E-01	-5.1108939E-01
87	198.567	4.3504341E 00	4.0662822E 02	-2.0503933E 02	7.9439611E-01	-3.2757463E-01	-5.1149741E-01
0	198.644	4.3516058E-04	4.0657312E 02	-2.0509491E 02	7.9419141E-01	-3.2775663E-01	-5.1169864E-01

TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS

SS PLATE EASTVILLE

X	Y	R	H	DH
176.84700	44.88945	4.3547645E 02	-2.3637424E 02	3.1999512E 00
186.71700	44.88481	4.2398649E 02	-2.4454113E 02	2.8698730E 00
199.26900	44.87892	4.1169173E 02	-2.5404575E 02	2.5107422E 00
202.44800	44.87743	4.0894455E 02	-2.5631151E 02	2.4289551E 00
202.52500	44.87739	4.0887970E 02	-2.5636548E 02	2.4272461E 00

TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS

SS PLATE EASTVILLE

DASH	X	Y	R	COSZ	H	D
0	176.84700	44.88945	4.3547645E 02	-5.5014271E-01	-2.3637424E 02	0.
1	186.71700	44.88481	4.2398649E 02	-5.8353510E-01	-2.4454113E 02	2.2231124E 01
2	186.97500	44.88469	4.2370886E 02	-5.8438210E-01	-2.4474630E 02	2.2789602E 01
3	187.36800	44.88451	4.2328805E 02	-5.8566974E-01	-2.4505740E 02	2.3638179E 01
4	187.66000	44.88437	4.2297705E 02	-5.8662445E-01	-2.4528810E 02	2.4267021E 01
5	187.96200	44.88423	4.2265689E 02	-5.8761002E-01	-2.4552613E 02	2.4915907E 01
6	188.29300	44.88407	4.2230767E 02	-5.8868817E-01	-2.4578624E 02	2.5625307E 01
7	188.61700	44.88392	4.2196758E 02	-5.8974135E-01	-2.4604045E 02	2.6310123E 01
50	201.70100	44.87770	4.0957764E 02	-6.3040320E-01	-2.5578382E 02	5.2927168E 01
52	202.22000	44.87753	4.0913699E 02	-6.3202673E-01	-2.5615870E 02	5.3930402E 01
53	202.44800	44.87743	4.0894455E 02	-6.3270305E-01	-2.5631151E 02	5.4370050E 01

TRAILBLAZER 1K OPTICAL TRAJECTORY SL-SS

SS PLATE EASTVILLE

XQ= 1.2665600E 02 YQ= 1.0333000E 01 XC= 1.2665600E 02 YC= 1.0333000E 01
F2= 4.0740000E 03 P= 1.0000000E-01 OCCULT= 2.0000000E 00

DASH	X	TIME	RANGE	HEIGHT	ALPHA	BETA	GAMMA
0	176.847	0.	4.3547645E 02	-2.3637424E 02	8.3382429E-01	4.5606073E-02	-5.5014271E-01
1	186.717	5.1287907E-02	4.2398649E 02	-2.4454113E 02	8.1164647E-01	2.6769668E-02	-5.8353510E-01
2	186.975	1.0131738E-01	4.2370886E 02	-2.4474630E 02	8.1105279E-01	2.6282631E-02	-5.8438210E-01
3	187.368	1.5136192E-01	4.2328805E 02	-2.4505740E 02	8.1014718E-01	2.5541331E-02	-5.8566974E-01
4	187.660	2.0139474E-01	4.2297705E 02	-2.4528810E 02	8.0947332E-01	2.4990983E-02	-5.8662444E-01
5	187.962	2.5142843E-01	4.2265689E 02	-2.4552613E 02	8.0877550E-01	2.4422187E-02	-5.8761004E-01
6	188.293	3.0146506E-01	4.2230767E 02	-2.4578624E 02	8.0800972E-01	2.3799252E-02	-5.8868817E-01
7	188.617	3.5150063E-01	4.2196758E 02	-2.4604045E 02	8.0725908E-01	2.3109973E-02	-5.8974136E-01
50	201.701	2.5027305E 00	4.0957764E 02	-2.5578382E 02	7.7628218E-01	-9.7881629E-04	-6.3040319E-01
52	202.220	2.6027721E 00	4.0913699E 02	-2.5615870E 02	7.7494413E-01	-1.9186363E-03	-6.3202673E-01
53	202.440	2.6527902E 00	4.0894455E 02	-2.5631151E 02	7.7439092E-01	-2.3310296E-03	-6.3270309E-01

```
*      WILSON,FLORENCE  PLATE CALIBRATION PROGRAM OF FEB. 18,1965
*      LIST
*      LABEL
*      SYMBOL TABLE

CPCAL           PLATE CALIBRATION PROGRAM
C
C               DIMENSION STATEMENTS
      DIMENSION XX(5),XY(5),XZ(5),YX(5),YY(5),YZ(5),ZX(5),ZY(5),ZZ(5),
     1X(25),Y(25),EL(25),EM(25),EN(25),COSSIG(25),EXI(25),ETA(25),TITLE(
     212)
C
C               COMMON STATEMENTS
      COMMON TITLE,EXI,ETA,X,Y,N
C
C               FORMAT STATEMENTS
   20 FORMAT(12A6)
   21 FORMAT(10I5)
   22 FORMAT(72H
     1                                   )
   23 FORMAT(3E15.8)
   24 FORMAT(1H 9F13.8)
   25 FORMAT(2F4.0,F8.4,2F4.0,F7.3,2F10.5,I5,E15.8)
   26 FORMAT(17HORT. ASC. CENTER=F4.0,2H HF4.0,2H MF8.4,2H S/13H DEC. CE
     1NTER=F4.0,2H DF4.0,2H MF7.3,2H S/4H XC=F10.5,7H    YC=F10.5/8H DTH
     2ETA=1PE15.7)
   27 FORMAT(72H0 RIGHT ASCENSION       DECLINATION            X
     1      Y            /37H HR   MIN    SEC        DEG  MIN   SEC)
   28 FORMAT(1H F4.0,F5.0,F9.4,F8.0,F5.0,F8.3,F13.3,F13.3)
   29 FORMAT(17HORT. ASC. CENTER=F4.0,2H HF4.0,2H MF8.4,2H S/13H DEC. CE
     1NTER=F4.0,2H DF4.0,2H MF7.3,2H S/6H0LAMC=1PE15.7,10H      MUC=1PE1
     25.7,10H       NUC=1PE15.7/8H LAMETA=1PE15.7,10H     MUETA=1PE15.7,10
     3H    NUETA=1PE15.7/8H LAMEXI=1PE15.7,10H     MUEXI=1PE15.7,10H     N
     4UEXI=1PE15.7)
   30 FORMAT(104H0NO.        LAMDA                MU                 NU
     1          COSSIG                ETA                XI)
   31 FORMAT(I4,1P6E18.7)
   32 FORMAT(3H A=1PE15.7,10H        B=1PE15.7,10H         C=1PE15.7,10H
     1        D=1PE15.7/6H ACAP=1PE15.7,10H      BCAP=1PE15.7,10H      CCAP
     2=1PE15.7,10H      DCAP=1PE15.7/7H STARS I5,2H  I5)
   33 FORMAT(72H0NO        X(S)        Y(S)     DX          DY
     1                                   )
   34 FORMAT(I3,2F10.3,2F10.5)
   35 FORMAT(1H 1P3E15.7)
   36 FORMAT(32H0CORRECTED COORDINATES OF CENTER)
   37 FORMAT(6H0LAMC=1PE15.7,10H       MUC=1PE15.7,10H       NUC=1PE15.7/8
     1H LAMETA=1PE15.7,10H     MUETA=1PE15.7,10H     NUETA=1PE15.7/8H LAME
     2XI=1PE15.7,10H     MUEXI=1PE15.7,10H     NUEXI=1PE15.7)
   38 FORMAT(6H0AEXI=1PE15.7,8H    BEXI=1PE15.7,8H    CEXI=1PE15.7/6H AETA
     1=1PE15.7,8H    BETA=1PE15.7,8H    CETA=1PE15.7)
C
C               DEFINE PI AND DTRAD=DEGREES TO RADIANS
      PI=3.141592654
      DTRAD=3.141592654/180.
C
C               READ AND WRITE TITLE
    1 READ INPUT TAPE 2,20,TITLE
      WRITE OUTPUT TAPE 3,20,TITLE
```

```
C             NUMBER OF SETS OF PRECESSION CONSTANTS AND NUMBER OF STARS
      READ INPUT TAPE 2,21,NN,N
C
C             READ IN FOUR CARDS FOR EACH PRECESSION SET
C             CARD ONE GIVES DATES FOR PRECESSION
C             CARDS TWO-FOUR GIVE PRECESSION CONSTANTS--THREE PER CARD
      DO 100 I=1,NN
      READ INPUT TAPE 2,22
      READ INPUT TAPE 2,23,XX(I),XY(I),XZ(I),YX(I),YY(I),YZ(I),ZX(I),ZY(
     1I),ZZ(I)
      WRITE OUTPUT TAPE 3,22
  100 WRITE OUTPUT TAPE 3,24,XX(I),XY(I),XZ(I),YX(I),YY(I),YZ(I),ZX(I),Z
     1Y(I),ZZ(I)
C
C             PLATE CENTER CALCULATIONS
C
C             READ AND WRITE PLATE CENTER DATA
      READ INPUT TAPE 2,25,ACHR,ACMIN,ACSEC,DCDEG,DCMIN,DCSEC,XC,YC,M,DT
     1H
      WRITE OUTPUT TAPE 3,26,ACHR,ACMIN,ACSEC,DCDEG,DCMIN,DCSEC,XC,YC,DT
     1H
C             COMPUTE RIGHT ASCENSION AND DECLINATION OF PLATE CENTER
C             IN DEGREES AND RADIANS
      AC=15.*(ACHR+(ACMIN+ACSEC/60.)/60.)
      DC=DCDEG+(DCMIN+DCSEC/60.)/60.
      AC=DTRAD*AC
      DC=DTRAD*DC
C             COMPUTE UNPRECESSED DIRECTION COSINES OF PLATE CENTER
C             EQ. 14
      IF(DC)60,61,62
   60 DC=-DC
      ENC=-SINF(DC)
      GO TO 63
   61 ENC=0.
      GO TO 63
   62 ENC=SINF(DC)
   63 ELC=COSF(DC)*COSF(AC)
      EMC=COSF(DC)*SINF(AC)
C
C             PRECESSION OF PLATE CENTER
C             M IS THE PRECESSION SET TO USE FOR PLATE CENTER
      I=M
C             PRECESS PLATE CENTER DIRECTION COSINES BY EQ. 5
      ELCP=XX(I)*ELC+XY(I)*EMC+XZ(I)*ENC
      EMCP=YX(I)*ELC+YY(I)*EMC+YZ(I)*ENC
      ENCP=ZX(I)*ELC+ZY(I)*EMC+ZZ(I)*ENC
C
C             COMPUTE DIRECTION COSINES OF XI AND ETA AXES--EQ. 15,16
      ENETA=SQRTF(1.-ENCP*ENCP)
      ELETA=-ENCP*ELCP/ENETA
      EMETA=-ENCP*EMCP/ENETA
      ELEXI=-EMCP/ENETA
      EMEXI=ELCP/ENETA
      ENEXI=0.
C
C             WRITE COLUMN HEADINGS
      WRITE OUTPUT TAPE 3,27
C
```

```
C             CALCULATIONS FOR STARS
C             PERFORM LOOP FOR EACH STAR
      DO 101 J=1,N
C             READ IN DATA FOR CURRENT STAR
      READ INPUT TAPE 2,25,AHR,AMIN,ASEC,DDEG,DMIN,DSEC,X(J),Y(J),M
      WRITE OUTPUT TAPE 3,28,AHR,AMIN,ASEC,DDEG,DMIN,DSEC,X(J),Y(J)
C             COMPUTE UNPRECESSED RIGHT ASCENSION AND DECLINATION
C             OF STAR IN DEGREES AND RADIANS
   65 A=15.*(AHR+(AMIN+ASEC/60.)/60.)
   66 D=DDEG+(DMIN+DSEC/60.)/60.
      A=DTRAD*A
      D=DTRAD*D
C             COMPUTE UNPRECESSED DIRECTION COSINES OF STAR --EQ. 4
      IF(D)67,68,69
   67 D=-D
      ENS=-SINF(D)
      GO TO 70
   68 ENS=0.
      GO TO 70
   69 ENS=SINF(D)
   70 ELS=COSF(D)*COSF(A)
      EMS=COSF(D)*SINF(A)
C
C             PRECESSION OF CURRENT STAR
C             M=PRECESSION SET TO USE WITH CURRENT STAR
      I=M
C             PRECESS DIRECTION COSINES OF CURRENT STAR--EQ. 5
      EL(J)=ELS*XX(I)+EMS*XY(I)+ENS*XZ(I)
      EM(J)=ELS*YX(I)+EMS*YY(I)+ENS*YZ(I)
      EN(J)=ELS*ZX(I)+EMS*ZY(I)+ENS*ZZ(I)
C
C             COMPUTE STANDARD COORDINATES OF CURRENT STAR--EQ. 18,19
      COSSIG(J)=EL(J)*ELCP+EM(J)*EMCP+EN(J)*ENCP
      EXI(J)=(EL(J)*ELEXI+EM(J)*EMEXI)/COSSIG(J)
  101 ETA(J)=(EL(J)*ELETA+EM(J)*EMETA+EN(J)*ENETA)/COSSIG(J)
C
C             WRITE OUT DATA COMPUTED FOR PLATE CENTER
      WRITE OUTPUT TAPE 3,20,TITLE
      WRITE OUTPUT TAPE 3,29,ACHR,ACMIN,ACSEC,DCDEG,DCMIN,DCSEC,ELCP,EMC
     1P,ENCP,ELETA,EMETA,ENETA,ELEXI,EMEXI,ENEXI
C             WRITE COLUMN HEADINGS
      WRITE OUTPUT TAPE 3,30
C
C             WRITE OUT COMPUTED DIRECTION COSINES AND STANDARD
C             COORDINATES FOR EACH STAR
      DO 102 J=1,N
  102 WRITE OUTPUT TAPE 3,31,J,EL(J),EM(J),EN(J),COSSIG(J),ETA(J),EXI(J)
C
C             NUMBER OF FOUR CONSTANT SOLUTIONS TO PERFORM
      READ INPUT TAPE 2,21,MM
C
C             LOOP FOR EACH FOUR CONSTANT SOLUTION
      DO 103 J=1,MM
C             READ IN STARS TO USE FOR END POINTS IN SOLUTION
      READ INPUT TAPE 2,21,L,LL
C
C             PERFORM FOUR CONSTANT SOLUTION
C             COMPUTE A,B,C,D FOR THESE TWO STARS--EQ. 20
```

```
      DEN=(ETA(LL)-ETA(L))**2+(EXI(LL)-EXI(L))**2
      IF(DEN-1.E-10)103,104,104
  104 A=(X(LL)-X(L))*(EXI(LL)-EXI(L))-(Y(LL)-Y(L))*(ETA(LL)-ETA(L))
      A=A/DEN
      B=(EXI(LL)-EXI(L))*(Y(LL)-Y(L))+(ETA(LL)-ETA(L))*(X(LL)-X(L))
      B=B/DEN
      C=X(LL)-A*EXI(LL)-B*ETA(LL)
      D=Y(LL)-B*EXI(LL)+A*ETA(LL)
      DEN=A*A+B*B
C           IF DEN TOO SMALL IGNORE THIS STAR
      IF(DEN-1.E-10)103,105,105
C
C           COMPUTE INVERSE PLATE CONSTANTS--EQ. 21
  105 ACAP=A/DEN
      BCAP=B/DEN
      CCAP=-(A*C+B*D)/DEN
      DCAP=(A*D-B*C)/DEN
      ECAP=-ACAP
C
C           WRITE OUT PLATE CONSTANTS AND INVERSE PLATE CONSTANTS
C           FROM FOUR CONSTANT SOLUTION
      WRITE OUTPUT TAPE 3,20,TITLE
      WRITE OUTPUT TAPE 3,32,A,B,C,D,ACAP,BCAP,CCAP,DCAP,L,LL
      WRITE OUTPUT TAPE 3,38,ACAP,BCAP,CCAP,BCAP,ECAP,DCAP
C
C           WRITE COLUMN HEADINGS
      WRITE OUTPUT TAPE 3,33
C
C           WRITE INVERSE PLATE CONSTANTS ON CARDS FOR USE IN
C           TRAJECTORY PROGRAM
      WRITE OUTPUT TAPE 14,20,TITLE
      WRITE OUTPUT TAPE 14,35,ACAP,BCAP,CCAP,BCAP,ECAP,DCAP
C
C           COMPUTE RESIDUALS FOR EACH STAR FOR FOUR CONSTANT WORK
C
      DO 106  I=1,N
C           COMPUTE DX AND DY FOR CURRENT STAR--EQ. 23
      DX=A*EXI(I)+B*ETA(I)+C-X(I)
      DY=B*EXI(I)-A*ETA(I)+D-Y(I)
C           WRITE OUT DX AND DY FOR CURRENT STAR
  106 WRITE OUTPUT TAPE 3,34,I,X(I),Y(I),DX,DY
  103 CONTINUE
C
C           PERFORM A SIX CONSTANT (LEAST SQUARES) SOLUTION
      CALL SIXCON
C
C           COMPUTE CORRECTED DIRECTION COSINES OF PLATE CENTER
C           AND AXES --EQ. 45
C           DTH READ IN AS SEC OF TIME MUST BE CONVERTED TO RADIANS
      DTH=DTH*PI/43200.
      SF=SINF(DTH)
      CF=COSF(DTH)
      ELC=ELCP*CF-EMCP*SF
      EMC=EMCP*CF+ELCP*SF
      ELEX=ELEXI*CF-EMEXI*SF
      EMEX=EMEXI*CF+ELEXI*SF
      ELET=ELETA*CF-EMETA*SF
      EMET=EMETA*CF+ELETA*SF
```

```
C
C              WRITE OUT CORRECTED DIRECTION COSINES
      WRITE OUTPUT TAPE 3,20,TITLE
      WRITE OUTPUT TAPE 3,36
      WRITE OUTPUT TAPE 3,37,ELC,EMC,ENCP,ELET,EMET,ENETA,ELEX,EMEX,ENEX
     1I
C              WRITE CORRECTED DIRECTION COSINES ON CARDS
  121 WRITE OUTPUT TAPE 14,35,ELC,ELEX,ELET,EMC,EMEX,EMET,ENCP,ENETA
C              DO NEXT SET OF DATA IF ANY
      GO TO 1
      END
```

```
*     LIST
*     LABEL
*     SYMBOL TABLE
      SUBROUTINE SIXCON

CSCON          SUBROUTINE FOR SIX CONSTANT SOLUTION
C
C
C              DIMENSION STATEMENTS
      DIMENSION TITLE(12),EXI(25),ETA(25),X(25),Y(25),A(10,10),B(10,10),
     1TWO(10)
C
      COMMON TITLE,EXI,ETA,X,Y,N
C
C              FORMAT STATEMENTS
   20 FORMAT(12A6)
   21 FORMAT(6H0AEXI=1PE15.7,8H   BEXI=1PE15.7,8H   CEXI=1PE15.7/6H AETA
     1=1PE15.7,8H   BETA=1PE15.7,8H   CETA=1PE15.7)
   22 FORMAT(4H0AX=1PE15.7,8H      BX=1PE15.7,8H      CX=1PE15.7/4H AY=1PE
     115.7,8H      BY=1PE15.7,8H      CY=1PE15.7)
   23 FORMAT(1H 1P3E15.7/1H 1P3E15.7)
   33 FORMAT(72H0NO        X(S)        Y(S)     DX         DY
     1              )
   34 FORMAT(I3,2F10.3,2F10.5)
   35 FORMAT(8H0MATRIX=I5)
C
C              SET ALL SUMS TO ZERO AT START
      ONE=1.
      SEXI2=0.
      SEXIET=0.
      SEXI=0.
      SEXEXI=0.
      SETA2=0.
      SETA=0.
      SEXETA=0.
      SEX=0.
      SEYEXI=0.
      SEYETA=0.
      SEY=0.
      EN=N
C
C              N=NUMBER OF STARS
C              PERFORM LOOP FOR EACH STAR
      DO 100 J=1,N
C
C              SUM OF XI SQUARED
      SEXI2=SEXI2+EXI(J)*EXI(J)
C              SUM OF XI TIMES ETA
      SEXIET=SEXIET+EXI(J)*ETA(J)
C              SUM OF XI
      SEXI=SEXI+EXI(J)
C              SUM OF X TIMES XI
      SEXEXI=SEXEXI+X(J)*EXI(J)
C              SUM OF ETA SQUARED
      SETA2=SETA2+ETA(J)*ETA(J)
C              SUM OF ETA
      SETA=SETA+ETA(J)
C              SUM OF X TIMES ETA
```

```
      SEXETA=SEXETA+X(J)*ETA(J)
C             SUM OF X
      SEX=SEX+X(J)
C             SUM OF Y TIMES XI
      SEYEXI=SEYEXI+Y(J)*EXI(J)
C             SUM OF Y TIMES ETA
      SEYETA=SEYETA+Y(J)*ETA(J)
C             SUM OF Y
  100 SEY=SEY+Y(J)
C
C             SET UP MATRICES AS IN EQ. 46
      A(1,1)=SEXI2
      A(1,2)=SEXIET
      A(1,3)=SEXI
      A(2,1)=SEXIET
      A(2,2)=SETA2
      A(2,3)=SETA
      A(3,1)=SEXI
      A(3,2)=SETA
      A(3,3)=EN
      B(1,1)=SEXEXI
      B(2,1)=SEXETA
      B(3,1)=SEX
      B(1,2)=SEYEXI
      B(2,2)=SEYETA
  200 B(3,2)=SEY
C
C             SOLVE MATRIX EQUATION BY LIBRARY ROUTINE XSIMEQF
      MATRIX=XSIMEQF(10,3,2,A,B,ONE,TWO)
C
      WRITE OUTPUT TAPE 3,20,TITLE
C             PRINT OUT CODE WORD=1 IF SOLUTION SUCCESSFUL
C             OTHERWISE IS GREATER THAN 1
      WRITE OUTPUT TAPE 3,35,MATRIX
C
C             WRITE OUT PLATE CONSTANTS FROM SIX CONSTANT SOLUTION
      WRITE OUTPUT TAPE 3,22,A(1,1),A(2,1),A(3,1),A(1,2),A(2,2),A(3,2)
C
C             COMPUTE INVERSE PLATE CONSTANTS--EQ. 23
      CXY=A(1,1)*A(2,2)-A(1,2)*A(2,1)
      ASEXI=A(2,2)/CXY
      BSEXI=-A(2,1)/CXY
      CSEXI=(-A(2,2)*A(3,1)+A(2,1)*A(3,2))/CXY
      ASETA=-A(1,2)/CXY
      BSETA=A(1,1)/CXY
      CSETA=(A(1,2)*A(3,1)-A(1,1)*A(3,2))/CXY
C
C             WRITE OUT INVERSE PLATE CONSTANTS
      WRITE OUTPUT TAPE 3,21,ASEXI,BSEXI,CSEXI,ASETA,BSETA,CSETA
C             PUT INVERSE PLATE CONSTANTS ON CARDS
      WRITE OUTPUT TAPE 14,23,ASEXI,BSEXI,CSEXI,ASETA,BSETA,CSETA
C
C             WRITE OUT COLUMN HEADINGS
      WRITE OUTPUT TAPE 3,33
C
C             COMPUTE AND WRITE OUT RESIDUALS FOR EACH STAR
C             COMPUTED BY SIX CONSTANT SOLUTION
      DO 101 J=1,N
```

```
      DX=A(1,1)*EXI(J)+A(2,1)*ETA(J)+A(3,1)-X(J)
      DY=A(1,2)*EXI(J)+A(2,2)*ETA(J)+A(3,2)-Y(J)
  101 WRITE OUTPUT TAPE 3,34,J,X(J),Y(J),DX,DY
C
C              RETURN TO MAIN PROGRAM
      RETURN
      END
```

```
*     WILSON,FLORENCE  OPTICAL TRAJECTORY   ASSEMBLY OF 2/17/65
*     LIST
*     LABEL
*     SYMBOL TABLE

CTRAJ         OPTICAL TRAJECTORY REDUCTION PROGRAM

C
C             DIMENSION STATEMENTS
      DIMENSION CAX(2),CAY(2),CBX(2),CBY(2),CCX(2),CCY(2),COSZ(250,2),CO
     1SZ1(5,2),CPHI(2),CPHIP(2),CTH(2),D(250,2),DLONG(2),EL(5,2),ELAZ(25
     20,2),ELC(2),ELETA(2),ELEV(2),ELEXI(2),ELON(2),ELONH(2),ELONM(2),EL
     3ONS(2),ELP(2),ELZ(2),EM(5,2),EMAZ(250,2),EMC(2),EMETA(2),EMEXI(2),
     4EMP(2),EMZ(2),EN(5,2),ENAZ(250,2),ENC(2),ENETA(2),ENP(2),ENZ(2),ET
     5AP(250,2),EXIP(250,2),F2(2),GTHR(2),GTM(2),GTS(2),H(5,2),HEIGHT(25
     60,2),IFEXT(2),IFPLUS(2),NDASH(250,2),NUM(2),NWT(250,2),OCCULT(2),P
     7(2),PEL(250,2),PEM(250,2),PEN(250,2),PHD(2),PHI(2),PHIP(2),PHM(2),
     8PHS(2),R(5,2),RANGE(250,2),RHO(2),SINL(2),SLOPE(2),SPHI(2),SPHIP(2
     9),STH(2),THETA(2),TIME(250,2),TITLE(12),TITLE1(12,2)
      DIMENSION X(5,2),XBAR(2),XC(2),XP(250,2),XQ(2),XTR(30),Y(5,2),YBAR
     1(2),YC(2),YP(250,2),YQ(2),YTR(30),ZETAP(250,2),ZTRAIL(2)
C
C             COMMON STATEMENTS
      COMMON CAX,CAY,CBX,CBY,CCX,CCY,CDL,COSZ,COSZ1,CPHI,CPHIP,CTH,D,DL,
     1DLONG,DTRAD,EL,ELAZ,ELC,ELETA,ELEV,ELEXI,ELON,ELONH,ELONM,ELONS,EL
     2P,ELR,ELZ,EM,EMAZ,EMC,EMETA,EMEXI,EMP,EMR,EMZ,EN,ENAZ,ENC,ENETA,EN
     3P,ENR,ENZ,ETAAB,ETAABO,ETAP,EXIAB,EXIABO,EXIP,F2,GTHR,GTM,GTS,H,HE
     4IGHT,IFEXT,IFPLUS,J,K,N,NDASH,NUM,NWT,OCCULT,P,PEL,PEM,PEN,PHD,PHI
     5,PHIP,PHM,PHS,PI,R,RANGE,RHO,SDL,SINL,SINQ,SLOPE,SPHI,SPHIP,STH,TC
     6ORR,THETA,THETOH,THETOM,THETOS,TIME,TITLE,TITLE1,X,XBAR,XC,XP,XQ,X
     7TR,XO,Y,YBAR,YC,YP,YQ,YTR,YO,ZETAAB,ZETAP,ZTRAIL
C
C             FORMAT STATEMENTS
   20 FORMAT(12A6)
   21 FORMAT(10I5)
   22 FORMAT(3(2F4.0,F8.4),F11.7)
   23 FORMAT(7HOTHETO=F4.0,1HHF4.0,1HMF8.4,1HS)
   24 FORMAT(6H TIME(I1,2H)=F4.0,1HHF4.0,1HMF8.4,2HS /5H PHI(I1,2H)=F4.0
     1,1HDF4.0,1HMF8.4,1HS/6H LONG(I1,2H)=F4.0,1HHF4.0,1HMF8.4,1HS/6H EL
     2EV(I1,2H)=F11.7,4H KFT)
   25 FORMAT(4E15.8)
   26 FORMAT(3E15.8)
   27 FORMAT(7F10.5)
   28 FORMAT(2I5,F10.5,2I5,F10.5,2I5,F10.5)
   29 FORMAT(38HOACCUMULATOR OVERFLOW AT END OF DASHPT)
   30 FORMAT(30HODIVIDE CHECK AT END OF DASHPT)
   31 FORMAT(27HONO CHECKS AT END OF DASHPT)
   32 FORMAT(21HOACCUMULATOR OVERFLOW)
   33 FORMAT(13HODIVIDE CHECK)
   35 FORMAT(1H 4E15.8)
   36 FORMAT(1H 3E15.8)
   37 FORMAT(1H 7F10.5)
   38 FORMAT(1H 2I5,F10.5,2I5,F10.5,2I5,F10.5)
C
C             TURN OFF INDICATORS AT START FOR LATER TESTING
    1 IF ACCUMULATOR OVERFLOW 10,10
   10 IF DIVIDE CHECK 11,11
C
```

```
C            DEFINE PI AND DTRAD=DEGREES TO RADIANS
   11 DTRAD=1.74532925E-2
      PI=3.141592654
C
C            READ AND WRITE TITLE
      READ INPUT TAPE 2,20,TITLE
      WRITE OUTPUT TAPE 3,20,TITLE
C            READ IN SIDEREAL TIME AT 0 HOURS Z FOR DATE OF EVENT
      READ INPUT TAPE 2,22,THETOH,THETOM,THETOS
      WRITE OUTPUT TAPE 3,23,THETOH,THETOM,THETOS
C
C            LOOP FOR READING IN DATA FROM EACH OF TWO STATIONS
      DO 100 J=1,2
      J=J
C            CURRENT STATION TITLE
      READ INPUT TAPE 2,20,(TITLE1(JL,J),JL=1,12)
      WRITE OUTPUT TAPE 3,20,(TITLE1(JL,J),JL=1,12)
C
C            DATA FOR COMPUTING RELATIVE COORDINATES
C            GT=GIVEN TIME (GREENWICH)---PH=LATITUDE OF STATION
C            ELON=LONGITUDE OF STATION---ELEV=ELEVATION OF STATION
      READ INPUT TAPE 2,22,GTHR(J),GTM(J),GTS(J),PHD(J),PHM(J),PHS(J),EL
     1ONH(J),ELONM(J),ELONS(J),ELEV(J)
      WRITE OUTPUT TAPE 3,24,J,GTHR(J),GTM(J),GTS(J),J,PHD(J),PHM(J),PHS
     1(J),J,ELONH(J),ELONM(J),ELONS(J),J,ELEV(J)
C
C            DATA FOR COMPUTING TRAIL EQUATION
C            N=NUMBER OF POINTS ALONG TRAIL--X0,Y0=APPROXIMATE X,Y
C            ALONG TRAIL---XTR,YTR=ADDITION TO X0,Y0 TO GIVE EXACT
C            READING
      READ INPUT TAPE 2,21,N
      READ INPUT TAPE 2,25,X0,Y0
      READ INPUT TAPE 2,25,(XTR(JJ),YTR(JJ),JJ=1,N)
      WRITE OUTPUT TAPE 3,35,X0,Y0
      WRITE OUTPUT TAPE 3,35,(XTR(JJ),YTR(JJ),JJ=1,N)
C            COMPUTE TRAIL EQUATION FOR STATION BY SUBROUTINE TRAIL
      CALL TRAIL
C
C            READ IN DATA TO USE IN COMPUTING POLE OF EVENT
C            DATA INCLUDES INVERSE PLATE CONSTANTS AND DIRECTION
C            COSINES OF PLATE CENTER AND COORDINATE AXES
C            DATA OBTAINED FROM PLATE CALIBRATION PROGRAM
   62 READ INPUT TAPE 2,26,CAX(J),CBX(J),CCX(J),CAY(J),CBY(J),CCY(J),ELC
     1(J),ELEXI(J),ELETA(J),EMC(J),EMEXI(J),EMETA(J),ENC(J),ENETA(J)
      WRITE OUTPUT TAPE 3,36,CAX(J),CBX(J),CCX(J),CAY(J),CBY(J),CCY(J),E
     1LC(J),ELEXI(J),ELETA(J),EMC(J),EMEXI(J),EMETA(J),ENC(J),ENETA(J)
C
C            READ IN X VALUES FOR FIVE POINTS OF SPECIAL INTEREST
      READ INPUT TAPE 2,27,(X(K,J),K=1,5)
      WRITE OUTPUT TAPE 3,37,(X(K,J),K=1,5)
C
C            READ IN DATA FOR EACH DASH
C            NUM=NUMBER OF DASHES READ AT THIS STATION
C            NDASH=NUMBER OF DASH---NWT=WEIGHT(QUALITY) OF THIS DASH
C            XP=ACTUAL X READING OF THIS DASH
C            THREE DASHES PER CARD
      READ INPUT TAPE 2,21,NUM(J)
      MUM=NUM(J)
```

```
      READ INPUT TAPE 2,28,(NDASH(K,J),NWT(K,J),XP(K,J),K=1,MUM)
      WRITE OUTPUT TAPE 3,38,(NDASH(K,J),NWT(K,J),XP(K,J),K=1,MUM)
C
C             READ IN DATA USED TO COMPUTE RELATIVE TIME
C             XQ,YQ=OBSERVED COORDINATES OF CENTER OF SHUTTER ROTATION
C             XC,YC=COORDINATES OF PROJECTION CENTER
C             F2=FOCAL LENGTH OF CAMERA SQUARED
C             IFPLUS=+1 FOR COUNTERCLOCKWISE ROTATION---IFPLUS=-1 FOR
C             CLOCKWISE ROTATION---IFPLUS=0 FOR NON SUPER SCHMIDT
C             P=PERIOD OF REVOLUTION OF SHUTTER---OCCULT=NUMBER OF
C             OCCULTATIONS PER REVOLUTION
C             IFEXT=+1 IF CARDS ARE TO BE MADE---IFEXT=0 IF NO CARDS
C             ARE TO BE MADE
  301 READ INPUT TAPE 2,25,XQ(J),YQ(J),XC(J),YC(J),F2(J),P(J),OCCULT(J)
      WRITE OUTPUT TAPE3,35,XQ(J),YQ(J),XC(J),YC(J),F2(J),P(J),OCCULT(J)
      READ INPUT TAPE 2,21,IFEXT(J),IFPLUS(J)
      WRITE OUTPUT TAPE 3,38,IFPLUS(J)
C             READ IN DATA FOR STATION 2 WHEN STATION 1 IS DONE
  100 CONTINUE
C
C             DATA ALL IN COMPUTER---START CALCULATIONS
C
C             WRITE TITLE ON NEW PAGE
      WRITE OUTPUT TAPE 3,20,TITLE
C
C             COMPUTE RELATIVE COORDINATES FOR THE TWO STATIONS
      CALL RELCO
C
C             COMPUTE THE POLE OF THE EVENT FOR EACH STATION
      DO 101 J=1,2
      J=J
      CALL POLE
  101 CONTINUE
C
C             COMPUTE THE RADIANT
      CALL RAD
C
C             PERFORM LOOP FOR EACH STATION
      DO 102 J=1,2
      J=J
C             COMPUTE RANGES AND HEIGHTS FOR THE FIVE POINTS OF
C             SPECIAL INTEREST FOR CURRENT STATION
      CALL RANDH
C             COMPUTE RANGES,HEIGHTS,COSINE OF ZENITH DIRECTION AND
C             DISTANCE ALONG THE TRAIL FOR EACH DASH OF CURRENT STATION
      CALL DASHPT
C             CHECK INDICATORS AT THIS POINT---IN CASE PROJECTION AND
C             SHUTTER ROTATION CENTERS VERY CLOSE WILL SET DIVIDE CHECK
C             IN NEXT SUBROUTINE----THIS WILL NOT HARM RESULTS
      IF ACCUMULATOR OVERFLOW 12,13
   12 WRITE OUTPUT TAPE 3,29
      IF DIVIDE CHECK 14,15
   14 WRITE OUTPUT TAPE 3,30
      GO TO 15
   13 IF DIVIDE CHECK 16,17
   16 WRITE OUTPUT TAPE 3,30
      GO TO 15
   17 WRITE OUTPUT TAPE 3,31
```

```
C
C             COMPUTE RELATIVE TIMES FOR THE CURRENT STATION FOR EACH DASH
C             DIRECTION COSINES IN ALTITUDE AZIMUTH SYSTEM ALSO
C             COMPUTED FOR EACH POINT IN SUBROUTINE RELTIM
   15 CALL RELTIM
C             COMPUTE FOR STATION 2 OR END PROGRAM HERE
  102 CONTINUE
C
C             CHECK INDICATORS AT END OF PROGRAM
      IF ACCUMULATOR OVERFLOW 200,201
  200 WRITE OUTPUT TAPE 3,32
  201 IF DIVIDE CHECK 202,203
  202 WRITE OUTPUT TAPE 3,33
  203 GO TO 1
      END
```

```
*     LIST
*     LABEL
*     SYMBOL TABLE
      SUBROUTINE TRAIL

CTRAIL LEAST SQUARES LINE THROUGH TRAIL  MAY 5,1964
C
C               DIMENSION STATEMENTS
      DIMENSION CAX(2),CAY(2),CBX(2),CBY(2),CCX(2),CCY(2),COSZ(250,2),CO
     1SZ1(5,2),CPHI(2),CPHIP(2),CTH(2),D(250,2),DLONG(2),EL(5,2),ELAZ(25
     20,2),ELC(2),ELETA(2),ELEV(2),ELEXI(2),ELON(2),ELONH(2),ELONM(2),EL
     3ONS(2),ELP(2),ELZ(2),EM(5,2),EMAZ(250,2),EMC(2),EMETA(2),EMEXI(2),
     4EMP(2),EMZ(2),EN(5,2),ENAZ(250,2),ENC(2),ENETA(2),ENP(2),ENZ(2),ET
     5AP(250,2),EXIP(250,2),F2(2),GTHR(2),GTM(2),GTS(2),H(5,2),HEIGHT(25
     60,2),IFEXT(2),IFPLUS(2),NDASH(250,2),NUM(2),NWT(250,2),OCCULT(2),P
     7(2),PEL(250,2),PEM(250,2),PEN(250,2),PHD(2),PHI(2),PHIP(2),PHM(2),
     8PHS(2),R(5,2),RANGE(250,2),RHO(2),SINL(2),SLOPE(2),SPHI(2),SPHIP(2
     9),STH(2),THETA(2),TIME(250,2),TITLE(12),TITLE1(12,2)
      DIMENSION X(5,2),XBAR(2),XC(2),XP(250,2),XQ(2),XTR(30),Y(5,2),YBAR
     1(2),YC(2),YP(250,2),YQ(2),YTR(30),ZETAP(250,2),ZTRAIL(2)
C
C
C               COMMON STATEMENTS
      COMMON CAX,CAY,CBX,CBY,CCX,CCY,CDL,COSZ,COSZ1,CPHI,CPHIP,CTH,D,DL,
     1DLONG,DTRAD,EL,ELAZ,ELC,ELETA,ELEV,ELEXI,ELON,ELONH,ELONM,ELONS,EL
     2P,ELR,ELZ,EM,EMAZ,EMC,EMETA,EMEXI,EMP,EMR,EMZ,EN,ENAZ,ENC,ENETA,EN
     3P,ENR,ENZ,ETAAB,ETAAB0,ETAP,EXIAB,EXIAB0,EXIP,F2,GTHR,GTM,GTS,H,HE
     4IGHT,IFEXT,IFPLUS,J,K,N,NDASH,NUM,NWT,OCCULT,P,PEL,PEM,PEN,PHD,PHI
     5,PHIP,PHM,PHS,PI,R,RANGE,RHO,SDL,SINL,SINQ,SLOPE,SPHI,SPHIP,STH,TC
     6ORR,THETA,THETOH,THETOM,THETOS,TIME,TITLE,TITLE1,X,XBAR,XC,XP,XQ,X
     7TR,X0,Y,YBAR,YC,YP,YQ,YTR,Y0,ZETAAB,ZETAP,ZTRAIL
C
C               ERROR MESSAGE FORMAT
   26 FORMAT(41H0DENOMINATOR LESS THAN 1.E-10 IN TRAIL EQ)
C
C               START COMPUTATION
C               N=NUMBER OF POINTS ON TRAIL
      EN=N
C               SET SUMS EQUAL TO ZERO AT START
      SUMX=0.
      SUMY=0.
      SUMXY=0.
      SUMX2=0.
C
C               COMPUTE SUMS OF POINTS ON TRAIL
      DO 100 JJ=1,N
C               SUM OF X
      SUMX=SUMX+XTR(JJ)
C               SUM OF Y
      SUMY=SUMY+YTR(JJ)
C               SUM OF X TIMES Y
      SUMXY=SUMXY+XTR(JJ)*YTR(JJ)
C               SUM OF X SQUARED
      SUMX2=SUMX2+XTR(JJ)*XTR(JJ)
  100 CONTINUE
C
C               COMPUTE AVERAGE X,AVERAGE Y AND SLOPE
      XBAR(J)=SUMX/EN
```

```
      YBAR(J)=SUMY/EN
C             COMPUTE SLOPE---EQ. 26
      SLOPE(J)=SUMXY-EN*XBAR(J)*YBAR(J)
      EMD=SUMX2-EN*XBAR(J)**2
C             IF DENOMINATOR TOO SMALL WRITE ERROR MESSAGE
      IF(EMD-1.E-10)92,91,91
   92 WRITE OUTPUT TAPE 3,26
   91 SLOPE(J)=SLOPE(J)/EMD
C             VALUE OF AVERAGE X IN MM
      XBAR(J)=XBAR(J)+X0
C             VALUE OF AVERAGE Y IN MM
  102 YBAR(J)=YBAR(J)+Y0
C             COMPUTE Y INTERCEPT---EQ. 25
      ZTRAIL(J)=YBAR(J)-SLOPE(J)*XBAR(J)
C             RETURN TO MAIN PROGRAM
      RETURN
      END
```

```
*     LIST
*     LABEL
*     SYMBOL TABLE
      SUBROUTINE RELCO

C  RC RELATIVE COORDINATES
C
C              DIMENSION STATEMENTS
       DIMENSION CAX(2),CAY(2),CBX(2),CBY(2),CCX(2),CCY(2),COSZ(250,2),CO
     1SZ1(5,2),CPHI(2),CPHIP(2),CTH(2),D(250,2),DLONG(2),EL(5,2),ELAZ(25
     20,2),ELC(2),ELETA(2),ELEV(2),ELEXI(2),ELON(2),ELONH(2),ELONM(2),EL
     3ONS(2),ELP(2),ELZ(2),EM(5,2),EMAZ(250,2),EMC(2),EMETA(2),EMEXI(2),
     4EMP(2),EMZ(2),EN(5,2),ENAZ(250,2),ENC(2),ENETA(2),ENP(2),ENZ(2),ET
     5AP(250,2),EXIP(250,2),F2(2),GTHR(2),GTM(2),GTS(2),H(5,2),HEIGHT(25
     60,2),IFEXT(2),IFPLUS(2),NDASH(250,2),NUM(2),NWT(250,2),OCCULT(2),P
     7(2),PEL(250,2),PEM(250,2),PEN(250,2),PHD(2),PHI(2),PHIP(2),PHM(2),
     8PHS(2),R(5,2),RANGE(250,2),RHO(2),SINL(2),SLOPE(2),SPHI(2),SPHIP(2
     9),STH(2),THETA(2),TIME(250,2),TITLE(12),TITLE1(12,2)
       DIMENSION X(5,2),XBAR(2),XC(2),XP(250,2),XQ(2),XTR(30),Y(5,2),YBAR
     1(2),YC(2),YP(250,2),YQ(2),YTR(30),ZETAP(250,2),ZTRAIL(2)
C
C              COMMON STATEMENTS
       COMMON CAX,CAY,CBX,CBY,CCX,CCY,CDL,COSZ,COSZ1,CPHI,CPHIP,CTH,D,DL,
     1DLONG,DTRAD,EL,ELAZ,ELC,ELETA,ELEV,ELEXI,ELON,ELONH,ELONM,ELONS,EL
     2P,ELR,ELZ,EM,EMAZ,EMC,EMETA,EMEXI,EMP,EMR,EMZ,EN,ENAZ,ENC,ENETA,EN
     3P,ENR,ENZ,ETAAB,ETAAB0,ETAP,EXIAB,EXIAB0,EXIP,F2,GTHR,GTM,GTS,H,HE
     4IGHT,IFEXT,IFPLUS,J,K,N,NDASH,NUM,NWT,OCCULT,P,PEL,PEM,PEN,PHD,PHI
     5,PHIP,PHM,PHS,PI,R,RANGE,RHO,SDL,SINL,SINQ,SLOPE,SPHI,SPHIP,STH,TC
     6ORR,THETA,THETOH,THETOM,THETOS,TIME,TITLE,TITLE1,X,XBAR,XC,XP,XQ,X
     7TR,XO,Y,YBAR,YC,YP,YQ,YTR,YO,ZETAAB,ZETAP,ZTRAIL
C
C              FORMAT STATEMENTS
   20 FORMAT(12A6)
   24 FORMAT(6H TIME(I1,2H)=F4.0,1HHF4.0,1HMF8.4,2HS /5H PHI(I1,2H)=F4.0
     1,1HDF4.0,1HMF8.4,1HS/6H LONG(I1,2H)=F4.0,1HHF4.0,1HMF8.4,1HS/6H EL
     2EV(I1,2H)=F11.7,4H KFT)
   26 FORMAT(7H THETA(I1,2H)=1PE15.7,4H DEG/5H ELZ(I1,2H)=1PE15.7,10H
     1    EMZ(I1,2H)=1PE15.7,10H        ENZ(I1,2H)=1PE15.7/10H PHIPRIME(I1,
     22H)=1PE15.7,4H DEG/5H RHO(I1,2H)=1PE15.7,4H KFT)
   27 FORMAT(8H0EXIAB0=1PE15.7,14H KFT    ETAAB0=1PE15.7,4H KFT/7H EXIAB=
     11PE15.7,13H KFT    ETAAB=1PE15.7,14H KFT    ZETAAB=1PE15.7,4H KFT/7H
     2 ERROR=1PE15.7,4H KFT)
C
C              COMPUTE SIDEREAL TIMES AND GEOGRAPHIC COORDINATES
C              FOR CURRENT STATION
      DO 101 J=1,2
      J=J
C              WRITE STATION TITLE AND INPUT COORDINATES
       WRITE OUTPUT TAPE 3,20,(TITLE1(JL,J),JL=1,12)
       WRITE OUTPUT TAPE 3,24,J,GTHR(J),GTM(J),GTS(J),J,PHD(J),PHM(J),PHS
     1(J),J,ELONH(J),ELONM(J),ELONS(J),J,ELEV(J)
C
C              COMPUTE UNIVERSAL TIME IN HOURS
      UT=GTHR(J)+(GTM(J)+GTS(J)/60.)/60.
C              COMPUTE CORRECTION IN THETA-- EQ. 9
      DTHETS=236.555*(UT/24.)
      DTHETS=DTHETS/3600.
C              COMPUTE THETAO IN HOURS
```

```
      TTHETO=THETOH+(THETOM+THETOS/60.)/60.
C             COMPUTE LONGITUDE OF STATION IN HOURS
      ELON(J)=ELONH(J)+(ELONM(J)+ELONS(J)/60.)/60.
C
C             COMPUTE SIDEREAL TIME IN DEGREES AND RADIANS--EQ. 10
      THET=15.*(TTHETO+UT-ELON(J)+DTHETS)
      THETA(J)=THET*DTRAD
      CTH(J)=COSF(THETA(J))
      STH(J)=SINF(THETA(J))
C
C             LATITUDE CALCULATIONS
      PHI(J)=(PHD(J)+(PHM(J)+PHS(J)/60.)/60.)*DTRAD
      SPHI(J)=SINF(PHI(J))
      CPHI(J)=COSF(PHI(J))
C             COMPUTE DIRECTION COSINES OF ZENITH DIRECTION--EQ. 31
      ELZ(J)=CPHI(J)*CTH(J)
      EMZ(J)=CPHI(J)*STH(J)
      ENZ(J)=SPHI(J)
C             COMPUTE GEOCENTRIC LATITUDE OF STATION--EQ. 28
   69 PHIP(J)=-695.6635*SINF(2.*PHI(J))+1.1731*SINF(4.*PHI(J))-.0026*SIN
     1F(6.*PHI(J))
   70 PHIP(J)=PHS(J)+PHIP(J)
      PHIPD=PHD(J)+(PHM(J)+PHIP(J)/60.)/60.
      PHIP(J)=PHIPD*DTRAD
C             COMPUTE EARTHS RADIUS OF STATION--EQ. 28
   71 RHO(J)=0.99832005+1.683494E-3*COSF(2.*PHI(J))-3.549E-6*COSF(4.*PHI
     1(J))+8.E-9*COSF(6.*PHI(J))
   72 RHO(J)=20.92642796E+3*RHO(J)
   73 RHO(J)=RHO(J)+ELEV(J)
      CPHIP(J)=COSF(PHIP(J))
      SPHIP(J)=SINF(PHIP(J))
C             LONGITUDE CALCULATIONS
      ELON(J)=15.*(ELONH(J)+(ELONM(J)+ELONS(J)/60.)/60.)*DTRAD
C
C             WRITE OUT SIDEREAL TIME,DIRECTION COSINES OF ZENITH
C             DIRECTION,GEOCENTRIC LATITUDE AND EARTHS RADIUS
  101 WRITE OUTPUT TAPE 3,26,J,THET,J,ELZ(J),J,EMZ(J),J,ENZ(J),J,PHIPD,J
     1,RHO(J)
C
C             CALCULATE LONGITUDE DIFFERENCE BETWEEN STATIONS
C             LONGITUDE MEASURED POSITIVE TOWARD WEST
   74 DL=ELON(1)-ELON(2)
C             IF VERY SMALL USE SERIES TO CALCULATE COS AND SIN
      IF(ABSF(DL-0.1))107,113,113
  107 CDL=COSF(ELON(1))*COSF(ELON(2))+SINF(ELON(1))*SINF(ELON(2))
      SDL=SINF(ELON(1))*COSF(ELON(2))-COSF(ELON(1))*SINF(ELON(2))
      GO TO 114
  113 IF(DL)108,109,110
  108 DL=-DL
      SDL=-SINF(DL)
      GO TO 111
  109 SDL=0.
      GO TO 111
  110 SDL=SINF(DL)
  111 CDL=COSF(DL)
C
C
C             CALCULATE DIRECTION COMPONENTS OF VECTOR FROM STATION 1
```

```
C             TO STATION 2 AT SIDEREAL TIME ZERO AT STATION 1--EQ. 29
  114 EXIABO=RHO(2)*CPHIP(2)*CDL-RHO(1)*CPHIP(1)
      ETAABO=RHO(2)*CPHIP(2)*SDL
      ZETAAB=RHO(2)*SPHIP(2)-RHO(1)*SPHIP(1)
C             RANGE BETWEEN STATION 1 AND STATION 2--EQ. 29
      RAB=SQRTF(EXIABO**2+ETAABO**2+ZETAAB**2)
C
C             COMPUTE DIRECTION COMPONENTS OF VECTOR FROM STATION 1
C             TO STATION 2 AT TIME OF EVENT--EQ. 30
      EXIAB=EXIABO*CTH(1)-ETAABO*STH(1)
      ETAAB=ETAABO*CTH(1)+EXIABO*STH(1)
      ERROR=SQRTF(EXIAB**2+ETAAB**2+ZETAAB**2)-RAB
C
C             WRITE OUT DIRECTION COMPONENTS OF THESE VECTORS
  112 WRITE OUTPUT TAPE 3,27,EXIABO,ETAABO,EXIAB,ETAAB,ZETAAB,ERROR
C             RETURN TO MAIN PROGRAM
      RETURN
      END
```

```
*     LIST
*     LABEL
*     SYMBOL TABLE
      SUBROUTINE POLE

C POLE CALCULATION OF POLE OF TRAIL
C
C             DIMENSION STATEMENTS
      DIMENSION CAX(2),CAY(2),CBX(2),CBY(2),CCX(2),CCY(2),COSZ(250,2),CO
     1SZ1(5,2),CPHI(2),CPHIP(2),CTH(2),D(250,2),DLONG(2),EL(5,2),ELAZ(25
     20,2),ELC(2),ELETA(2),ELEV(2),ELEXI(2),ELON(2),ELONH(2),ELONM(2),EL
     3ONS(2),ELP(2),ELZ(2),EM(5,2),EMAZ(250,2),EMC(2),EMETA(2),EMEXI(2),
     4EMP(2),EMZ(2),EN(5,2),ENAZ(250,2),ENC(2),ENETA(2),ENP(2),ENZ(2),ET
     5AP(250,2),EXIP(250,2),F2(2),GTHR(2),GTM(2),GTS(2),H(5,2),HEIGHT(25
     60,2),IFEXT(2),IFPLUS(2),NDASH(250,2),NUM(2),NWT(250,2),OCCULT(2),P
     7(2),PEL(250,2),PEM(250,2),PEN(250,2),PHD(2),PHI(2),PHIP(2),PHM(2),
     8PHS(2),R(5,2),RANGE(250,2),RHO(2),SINL(2),SLOPE(2),SPHI(2),SPHIP(2
     9),STH(2),THETA(2),TIME(250,2),TITLE(12),TITLE1(12,2)
      DIMENSION X(5,2),XBAR(2),XC(2),XP(250,2),XQ(2),XTR(30),Y(5,2),YBAR
     1(2),YC(2),YP(250,2),YQ(2),YTR(30),ZETAP(250,2),ZTRAIL(2)
C
C             COMMON STATEMENTS
      COMMON CAX,CAY,CBX,CBY,CCX,CCY,CDL,COSZ,COSZ1,CPHI,CPHIP,CTH,D,DL,
     1DLONG,DTRAD,EL,ELAZ,ELC,ELETA,ELEV,ELEXI,ELON,ELONH,ELONM,ELONS,EL
     2P,ELR,ELZ,EM,EMAZ,EMC,EMETA,EMEXI,EMP,EMR,EMZ,EN,ENAZ,ENC,ENETA,EN
     3P,ENR,ENZ,ETAAB,ETAABO,ETAP,EXIAB,EXIABO,EXIP,F2,GTHR,GTM,GTS,H,HE
     4IGHT,IFEXT,IFPLUS,J,K,N,NDASH,NUM,NWT,OCCULT,P,PEL,PEM,PEN,PHD,PHI
     5,PHIP,PHM,PHS,PI,R,RANGE,RHO,SDL,SINL,SINQ,SLOPE,SPHI,SPHIP,STH,TC
     6ORR,THETA,THETOH,THETOM,THETOS,TIME,TITLE,TITLE1,X,XBAR,XC,XP,XQ,X
     7TR,XO,Y,YBAR,YC,YP,YQ,YTR,YO,ZETAAB,ZETAP,ZTRAIL
C
C             FORMAT STATEMENTS
   20 FORMAT(12A6)
   29 FORMAT(5HOCAX=1PE15.7,7H   CBX=1PE15.7,7H   CCX=1PE15.7/5H CAY=1PE
     115.7,7H   CBY=1PE15.7,7H   CCY=1PE15.7/5H ELC=1PE15.7,9H   ELEXI=1
     2PE15.7,9H   ELETA=1PE15.7/5H EMC=1PE15.7,9H   EMEXI=1PE15.7,9H   E
     3META=1PE15.7/5H ENC=1PE15.7,9H   ENETA=1PE15.7)
   31 FORMAT(3HOY=1PE15.7,5H X  +1PE15.7)
   32 FORMAT(98HO     X                Y                    EL
     1EM                EN                ERROR)
   33 FORMAT(1H F10.5,F15.5,1P4E20.7)
   34 FORMAT(19H SINL LESS THAN E-5)
   35 FORMAT(5HOELP=1PE15.7,10H       EMP=1PE15.7,10H       ENP=1PE15.7/6H
     1 SINL=1PE15.7/14H TRAIL LENGTH=1PE15.7,4H DEG/7HOERROR=1PE15.7,3H
     2  1PE15.7,3H   1PE15.7)
C
C             WRITE OUT TITLE, INVERSE PLATE CONSTANTS, AND DIRECTION
C             COSINES OF PLATE CENTER AND COORDINATE AXES
      WRITE OUTPUT TAPE 3,20,TITLE
      WRITE OUTPUT TAPE 3,20,(TITLE1(JL,J),JL=1,12)
   63 WRITE OUTPUT TAPE 3,29,CAX(J),CBX(J),CCX(J),CAY(J),CBY(J),CCY(J),E
     1LC(J),ELEXI(J),ELETA(J),EMC(J),EMEXI(J),EMETA(J),ENC(J),ENETA(J)
C             WRITE OUT TRAIL EQUATION
      WRITE OUTPUT TAPE 3,31,SLOPE(J),ZTRAIL(J)
C             WRITE HEADS OF COLUMNS
      WRITE OUTPUT TAPE 3,32
C
C             CALCULATION OF STANDARD COORDINATES AND DIRECTION
```

```
C             COSINES FOR THE FIVE POINTS
C             PERFORM LOOP FOR THE FIVE POINTS
C             J=NUMBER OF STATION(I.E. 1 OR 2)---K=NUMBER OF POINT
C             READ AT THIS STATION
      DO 201  K=1,5
C             COMPUTE Y---EQ. 25
      Y(K,J)=SLOPE(J)*X(K,J)+ZTRAIL(J)
C             COMPUTE XIBAR AND ETABAR FOR GIVEN POINT--EQ. 23
      EXIBAR=CAX(J)*X(K,J)+CBX(J)*Y(K,J)+CCX(J)
      ETABAR=CAY(J)*X(K,J)+CBY(J)*Y(K,J)+CCY(J)
C             COMPUTE DIRECTION COSINES OF THIS POINT--EQ. 24
      DENOM=SQRTF(1.+EXIBAR**2+ETABAR**2)
      EL(K,J)=(ELC(J)+ELETA(J)*ETABAR+ELEXI(J)*EXIBAR)/DENOM
      EM(K,J)=(EMC(J)+EMETA(J)*ETABAR+EMEXI(J)*EXIBAR)/DENOM
      EN(K,J)=(ENC(J)+ENETA(J)*ETABAR)/DENOM
C             SUM OF DIRECTION COSINES SQUARED MUST BE ONE
      ERR1=EL(K,J)**2+EM(K,J)**2+EN(K,J)**2-1.
C
C             WRITE OUT X,Y AND DIRECTION COSINES OF THE POINT
  201 WRITE OUTPUT TAPE 3,33,X(K,J),Y(K,J),EL(K,J),EM(K,J),EN(K,J),ERR1
C
C             COMPUTE POLE OF TRAIL--EQ. 27
      C1=EM(1,J)*EN(5,J)-EN(1,J)*EM(5,J)
      C2=EN(1,J)*EL(5,J)-EL(1,J)*EN(5,J)
      C3=EL(1,J)*EM(5,J)-EM(1,J)*EL(5,J)
      SINL(J)=SQRTF(C1**2+C2**2+C3**2)
      ASINL=ABSF(SINL(J))
      IF(ASINL-1.E-5)202,203,203
  202 WRITE OUTPUT TAPE 3,34
  203 IF(ASINL-1.)204,204,205
  205 SINL(J)=1.
  204 ELP(J)=C1/SINL(J)
      EMP(J)=C2/SINL(J)
      ENP(J)=C3/SINL(J)
C             COMPUTE ERRORS
      ERRP=ELP(J)**2+EMP(J)**2+ENP(J)**2-1.
      ERRP1=ELP(J)*EL(1,J)+EMP(J)*EM(1,J)+ENP(J)*EN(1,J)
      ERRP2=ELP(J)*EL(5,J)+EMP(J)*EM(5,J)+ENP(J)*EN(5,J)
C             COMPUTE TRAIL LENGTH
      DLONG(J)=ASINF(SINL(J))/DTRAD
C
C             WRITE OUT DIRECTION COSINES OF POLE,SIN TRAIL LENGTH
C             AND LENGTH OF TRAIL
      WRITE OUTPUT TAPE 3,35,ELP(J),EMP(J),ENP(J),SINL(J),DLONG(J),ERRP,
     1ERRP1,ERRP2
C             RETURN TO MAIN PROGRAM
      RETURN
      END
```

```
*     LIST
*     LABEL
*     SYMBOL TABLE
      SUBROUTINE RAD
C RAD CALCULATION OF RADIANT
C
C             DIMENSION STATEMENTS
      DIMENSION CAX(2),CAY(2),CBX(2),CBY(2),CCX(2),CCY(2),COSZ(250,2),CO
     1SZ1(5,2),CPHI(2),CPHIP(2),CTH(2),D(250,2),DLONG(2),EL(5,2),ELAZ(25
     20,2),ELC(2),ELETA(2),ELEV(2),ELEXI(2),ELON(2),ELONH(2),ELONM(2),EL
     3ONS(2),ELP(2),ELZ(2),EM(5,2),EMAZ(250,2),EMC(2),EMETA(2),EMEXI(2),
     4EMP(2),EMZ(2),EN(5,2),ENAZ(250,2),ENC(2),ENETA(2),ENP(2),ENZ(2),ET
     5AP(250,2),EXIP(250,2),F2(2),GTHR(2),GTM(2),GTS(2),H(5,2),HEIGHT(25
     60,2),IFEXT(2),IFPLUS(2),NDASH(250,2),NUM(2),NWT(250,2),OCCULT(2),P
     7(2),PEL(250,2),PEM(250,2),PEN(250,2),PHD(2),PHI(2),PHIP(2),PHM(2),
     8PHS(2),R(5,2),RANGE(250,2),RHO(2),SINL(2),SLOPE(2),SPHI(2),SPHIP(2
     9),STH(2),THETA(2),TIME(250,2),TITLE(12),TITLE1(12,2)
      DIMENSION X(5,2),XBAR(2),XC(2),XP(250,2),XQ(2),XTR(30),Y(5,2),YBAR
     1(2),YC(2),YP(250,2),YQ(2),YTR(30),ZETAP(250,2),ZTRAIL(2)
C
C             COMMON STATEMENTS
      COMMON CAX,CAY,CBX,CBY,CCX,CCY,CDL,COSZ,COSZ1,CPHI,CPHIP,CTH,D,DL,
     1DLONG,DTRAD,EL,ELAZ,ELC,ELETA,ELEV,ELEXI,ELON,ELONH,ELONM,ELONS,EL
     2P,ELR,ELZ,EM,EMAZ,EMC,EMETA,EMEXI,EMP,EMR,EMZ,EN,ENAZ,ENC,ENETA,EN
     3P,ENR,ENZ,ETAAB,ETAABO,ETAP,EXIAB,EXIABO,EXIP,F2,GTHR,GTM,GTS,H,HE
     4IGHT,IFEXT,IFPLUS,J,K,N,NDASH,NUM,NWT,OCCULT,P,PEL,PEM,PEN,PHD,PHI
     5,PHIP,PHM,PHS,PI,R,RANGE,RHO,SDL,SINL,SINQ,SLOPE,SPHI,SPHIP,STH,TC
     6ORR,THETA,THETOH,THETOM,THETOS,TIME,TITLE,TITLE1,X,XBAR,XC,XP,XQ,X
     7TR,XO,Y,YBAR,YC,YP,YQ,YTR,YO,ZETAAB,ZETAP,ZTRAIL
C
C             FORMAT STATEMENTS
   20 FORMAT(12A6)
   37 FORMAT(8HORADIANT)
   38 FORMAT(5HOELR=1PE15.7,7H   EMR=1PE15.7,7H   ENR=1PE15.7/6H SINQ=1P
     1E15.7/7H ERROR=1PE15.7)
C
C             WRITE OUT TITLE
      WRITE OUTPUT TAPE 3,20,TITLE
C
C             COMPUTE RADIANT---EQ. 32
      C4=EMP(1)*ENP(2)-ENP(1)*EMP(2)
      C5=ENP(1)*ELP(2)-ELP(1)*ENP(2)
      C6=ELP(1)*EMP(2)-EMP(1)*ELP(2)
C             COMPUTE SIN Q---EQ. 32
      SINQ=SQRTF(C4**2+C5**2+C6**2)
      ELR=C4/SINQ
      EMR=C5/SINQ
   50 ENR=C6/SINQ
      ERROR=ELR**2+EMR**2+ENR**2-1.+ELR*ELP(1)+EMR*EMP(1)+ENR*ENP(1)+ELR
     1*ELP(2)+EMR*EMP(2)+ENR*ENP(2)
C
C             IS RADIANT ABOVE THE HORIZON
      IF(ELZ(1)*ELR+EMZ(1)*EMR+ENZ(1)*ENR)7,8,8
C             NO, CHANGE SIGNS SO THAT IT IS ABOVE HORIZON
    7 ELR=-ELR
      EMR=-EMR
      ENR=-ENR
```

```
C
    8 WRITE OUTPUT TAPE 3,37
C             WRITE OUT DIRECTION COSINES OF RADIANT AND SIN Q
    9 WRITE OUTPUT TAPE 3,38,ELR,EMR,ENR,SINQ,ERROR
C             RETURN TO MAIN PROGRAM
      RETURN
      END
```

```
*     LIST
*     LABEL
*     SYMBOL TABLE
      SUBROUTINE RANDH

CRANDH  RANGES AND HEIGHTS FOR THE FIVE POINTS
C
C                 DIMENSION STATEMENTS
      DIMENSION CAX(2),CAY(2),CBX(2),CBY(2),CCX(2),CCY(2),COSZ(250,2),CO
     1SZ1(5,2),CPHI(2),CPHIP(2),CTH(2),D(250,2),DLONG(2),EL(5,2),ELAZ(25
     20,2),ELC(2),ELETA(2),ELEV(2),ELEXI(2),ELON(2),ELONH(2),ELONM(2),EL
     3ONS(2),ELP(2),ELZ(2),EM(5,2),EMAZ(250,2),EMC(2),EMETA(2),EMEXI(2),
     4EMP(2),EMZ(2),EN(5,2),ENAZ(250,2),ENC(2),ENETA(2),ENP(2),ENZ(2),ET
     5AP(250,2),EXIP(250,2),F2(2),GTHR(2),GTM(2),GTS(2),H(5,2),HEIGHT(25
     60,2),IFEXT(2),IFPLUS(2),NDASH(250,2),NUM(2),NWT(250,2),OCCULT(2),P
     7(2),PEL(250,2),PEM(250,2),PEN(250,2),PHD(2),PHI(2),PHIP(2),PHM(2),
     8PHS(2),R(5,2),RANGE(250,2),RHO(2),SINL(2),SLOPE(2),SPHI(2),SPHIP(2
     9),STH(2),THETA(2),TIME(250,2),TITLE(12),TITLE1(12,2)
      DIMENSION X(5,2),XBAR(2),XC(2),XP(250,2),XQ(2),XTR(30),Y(5,2),YBAR
     1(2),YC(2),YP(250,2),YQ(2),YTR(30),ZETAP(250,2),ZTRAIL(2)
C
C                 COMMON STATEMENTS
      COMMON CAX,CAY,CBX,CBY,CCX,CCY,CDL,COSZ,COSZ1,CPHI,CPHIP,CTH,D,DL,
     1DLONG,DTRAD,EL,ELAZ,ELC,ELETA,ELEV,ELEXI,ELON,ELONH,ELONM,ELONS,EL
     2P,ELR,ELZ,EM,EMAZ,EMC,EMETA,EMEXI,EMP,EMR,EMZ,EN,ENAZ,ENC,ENETA,EN
     3P,ENR,ENZ,ETAAB,ETAAB0,ETAP,EXIAB,EXIAB0,EXIP,F2,GTHR,GTM,GTS,H,HE
     4IGHT,IFEXT,IFPLUS,J,K,N,NDASH,NUM,NWT,OCCULT,P,PEL,PEM,PEN,PHD,PHI
     5,PHIP,PHM,PHS,PI,R,RANGE,RHO,SDL,SINL,SINQ,SLOPE,SPHI,SPHIP,STH,TC
     6ORR,THETA,THETOH,THETOM,THETOS,TIME,TITLE,TITLE1,X,XBAR,XC,XP,XQ,X
     7TR,X0,Y,YBAR,YC,YP,YQ,YTR,Y0,ZETAAB,ZETAP,ZTRAIL

C                 FORMAT STATEMENTS
   20 FORMAT(12A6)
   33 FORMAT(1H F10.5,F15.5,1P3E20.7)
   41 FORMAT(77H0      X                 Y                    R
     1H                   DH)
C
C                 WRITE OUT TITLES AND COLUMN HEADINGS
      WRITE OUTPUT TAPE 3,20,TITLE
      WRITE OUTPUT TAPE 3,20,(TITLE1(JL,J),JL=1,12)
   62 WRITE OUTPUT TAPE 3,41
C
C                 COMPUTE DISTANCE FROM STATION TO PLANE OF EVENT
C                 IS THIS STATION 1
      IF(J-1)206,207,206
C                 NO COMPUTE DISTANCE FROM STATION 2 TO PLANE DETERMINED
C                 BY TRAIL OF EVENT AND STATION 1---EQ. 33
  206 L=1
      SST=-(EXIAB*ELP(L)+ETAAB*EMP(L)+ZETAAB*ENP(L))
      GO TO 208
C                 YES COMPUTE DISTANCE FROM STATION 1 TO PLANE DETERMINED
C                 BY TRAIL OF EVENT AND STATION 2---EQ. 33
  207 L=2
      SST=EXIAB*ELP(L)+ETAAB*EMP(L)+ZETAAB*ENP(L)
C
C                 COMPUTE RANGES,HEIGHTS AND COSINE OF ZENITH FOR EACH
C                 POINT
C                 J=NUMBER OF STATION--K=NUMBER OF POINT AT STATION
```

```
  208 DO 205 K=1,5
C             COMPUTE COS EPSILON---EQ. 34
      SK=EL(K,J)*ELP(L)+EM(K,J)*EMP(L)+EN(K,J)*ENP(L)
C             COMPUTE RANGE OF POINT FROM STATION--EQ. 34
      R(K,J)=SST/SK
C             COMPUTE COSINE OF ZENITH DISTANCE OF POINT--EQ. 35
      COSZ1(K,J)=ELZ(J)*EL(K,J)+EMZ(J)*EM(K,J)+ENZ(J)*EN(K,J)
C             COMPUTE HEIGHT ABOVE SEA LEVEL OF POINT---EQ. 38
      H1=R(K,J)*COSZ1(K,J)
  210 DH=SQRTF(RHO(J)*RHO(J)+R(K,J)*R(K,J)+2.*RHO(J)*H1)-(RHO(J)+H1)
      H(K,J)=H1+DH+ELEV(J)
C
C             WRITE OUT RANGE,HEIGHT, AND DH FOR EACH POINT
  205 WRITE OUTPUT TAPE 3,33,X(K,J),Y(K,J),R(K,J),H(K,J),DH
C
C             RETURN TO MAIN PROGRAM
      RETURN
      END
```

```
*     LIST
*     LABEL
*     SYMBOL TABLE
      SUBROUTINE DASHPT

CDASHPT  RANGES AND HEIGHTS FOR THE DASHES
C
C             DIMENSION STATEMENTS
      DIMENSION CAX(2),CAY(2),CBX(2),CBY(2),CCX(2),CCY(2),COSZ(250,2),CO
     1SZ1(5,2),CPHI(2),CPHIP(2),CTH(2),D(250,2),DLONG(2),EL(5,2),ELAZ(25
     20,2),ELC(2),ELETA(2),ELEV(2),ELEXI(2),ELON(2),ELONH(2),ELONM(2),EL
     3ONS(2),ELP(2),ELZ(2),EM(5,2),EMAZ(250,2),EMC(2),EMETA(2),EMEXI(2),
     4EMP(2),EMZ(2),EN(5,2),ENAZ(250,2),ENC(2),ENETA(2),ENP(2),ENZ(2),ET
     5AP(250,2),EXIP(250,2),F2(2),GTHR(2),GTM(2),GTS(2),H(5,2),HEIGHT(25
     60,2),IFEXT(2),IFPLUS(2),NDASH(250,2),NUM(2),NWT(250,2),OCCULT(2),P
     7(2),PEL(250,2),PEM(250,2),PEN(250,2),PHD(2),PHI(2),PHIP(2),PHM(2),
     8PHS(2),R(5,2),RANGE(250,2),RHO(2),SINL(2),SLOPE(2),SPHI(2),SPHIP(2
     9),STH(2),THETA(2),TIME(250,2),TITLE(12),TITLE1(12,2)
      DIMENSION X(5,2),XBAR(2),XC(2),XP(250,2),XQ(2),XTR(30),Y(5,2),YBAR
     1(2),YC(2),YP(250,2),YQ(2),YTR(30),ZETAP(250,2),ZTRAIL(2)
C
C             COMMON STATEMENTS
      COMMON CAX,CAY,CBX,CBY,CCX,CCY,CDL,COSZ,COSZ1,CPHI,CPHIP,CTH,D,DL,
     1DLONG,DTRAD,EL,ELAZ,ELC,ELETA,ELEV,ELEXI,ELON,ELONH,ELONM,ELONS,EL
     2P,ELR,ELZ,EM,EMAZ,EMC,EMETA,EMEXI,EMP,EMR,EMZ,EN,ENAZ,ENC,ENETA,EN
     3P,ENR,ENZ,ETAAB,ETAAB0,ETAP,EXIAB,EXIAB0,EXIP,F2,GTHR,GTM,GTS,H,HE
     4IGHT,IFEXT,IFPLUS,J,K,N,NDASH,NUM,NWT,OCCULT,P,PEL,PEM,PEN,PHD,PHI
     5,PHIP,PHM,PHS,PI,R,RANGE,RHO,SDL,SINL,SINQ,SLOPE,SPHI,SPHIP,STH,TC
     6ORR,THETA,THETOH,THETOM,THETOS,TIME,TITLE,TITLE1,X,XBAR,XC,XP,XQ,X
     7TR,X0,Y,YBAR,YC,YP,YQ,YTR,Y0,ZETAAB,ZETAP,ZTRAIL
C
C             FORMAT STATEMENTS
   20 FORMAT(12A6)
   24 FORMAT(1H I5,2F15.5,1P4E20.7)
   25 FORMAT(106H0 DASH             X                Y                R
     1              COSZ                H                  D)
C
C             MUM=NUMBER OF POINTS OR DASHES TO PROCESS
      MUM=NUM(J)
C             WRITE OUT TITLE AND COLUMN HEADINGS
      WRITE OUTPUT TAPE 3,20,TITLE
      WRITE OUTPUT TAPE 3,20,(TITLE1(JL,J),JL=1,12)
   62 WRITE OUTPUT TAPE 3,25
C
C             IS THIS STATION 1
      IF(J-1)102,103,102
C             NO COMPUTE DISTANCE FROM STATION 2 TO PLANE DETERMINED
C             BY TRAIL OF EVENT AND STATION 1--EQ. 33
  102 L=1
      SST=-(EXIAB*ELP(L)+ETAAB*EMP(L)+ZETAAB*ENP(L))
      GO TO 104
C             YES COMPUTE DISTANCE FROM STATION 1 TO PLANE DETERMINED
C             BY TRAIL OF EVENT AND STATION 2--EQ. 33
  103 L=2
      SST=EXIAB*ELP(L)+ETAAB*EMP(L)+ZETAAB*ENP(L)
C             SET LINE COUNT TO 1
  104 M=1
C
```

```
C             COMPUTE RANGE,HEIGHT,DIRECTION COSINES AND DISTANCES
C             ALONG THE TRAIL FOR EACH DASH
C             J=NUMBER OF STATION---K=NUMBER OF POINT AT STATION
      DO 101 K=1,MUM
C             COMPUTE Y---EQ. 25
      YP(K,J)=SLOPE(J)*XP(K,J)+ZTRAIL(J)
C             COMPUTE ETABAR AND XIBAR---EQ. 23
      EXIBAR=CAX(J)*XP(K,J)+CBX(J)*YP(K,J)+CCX(J)
      ETABAR=CAY(J)*XP(K,J)+CBY(J)*YP(K,J)+CCY(J)
C             COMPUTE DIRECTION COSINES---EQ. 24
      DENOM=SQRTF(1.+EXIBAR**2+ETABAR**2)
      PEL(K,J)=(ELC(J)+ELETA(J)*ETABAR+ELEXI(J)*EXIBAR)/DENOM
      PEM(K,J)=(EMC(J)+EMETA(J)*ETABAR+EMEXI(J)*EXIBAR)/DENOM
      PEN(K,J)=(ENC(J)+ENETA(J)*ETABAR)/DENOM
C             COMPUTE COS EPSILON---EQ. 34
      SK=PEL(K,J)*ELP(L)+PEM(K,J)*EMP(L)+PEN(K,J)*ENP(L)
C             COMPUTE RANGE FROM STATION---EQ. 34
      RANGE(K,J)=SST/SK
C             COMPUTE ZENITH DIRECTION COSINE---EQ. 35
      COSZ(K,J)=ELZ(J)*PEL(K,J)+EMZ(J)*PEM(K,J)+ENZ(J)*PEN(K,J)
C             COMPUTE HEIGHT ABOVE SEA LEVEL---EQ. 38
      H1=RANGE(K,J)*COSZ(K,J)
      DH=SQRTF(RHO(J)*RHO(J)+RANGE(K,J)*RANGE(K,J)+2.*RHO(J)*H1)-(RHO(J)
     1+H1)
      HEIGHT(K,J)=H1+DH+ELEV(J)
C             COMPUTE DIRECTION COMPONENTS OF POINT---EQ. 39
      EXIP(K,J)=RANGE(K,J)*PEL(K,J)
      ETAP(K,J)=RANGE(K,J)*PEM(K,J)
      ZETAP(K,J)=RANGE(K,J)*PEN(K,J)
  111 DD1=(EXIP(K,J)-EXIP(1,J))**2+(ETAP(K,J)-ETAP(1,J))**2+(ZETAP(K,J)-
     1ZETAP(1,J))**2
      DD1=SQRTF(DD1)
C
C             COMPUTE DISTANCE OF POINT FROM FIRST POINT ON TRAIL--EQ. 40
C             IS THIS THE FIRST POINT
      IF(K-1)105,106,105
C             YES,DISTANCE OF FIRST POINT
  106 D1=DD1
      D(K,J)=D1
      GO TO 107
C             NO COMPUTE DISTANCE FROM FIRST POINT
  105 D(K,J)=DD1-D1
C
C             WRITE OUT DASH NUMBER,X,Y,RANGE,COSINE ZENITH DIRECTION,
C             HEIGHT, AND DISTANCE ALONG THE TRAIL
  107 WRITE OUTPUT TAPE 3,24,NDASH(K,J),XP(K,J),YP(K,J),RANGE(K,J),COSZ(
     1K,J),HEIGHT(K,J),D(K,J)
C             INCREMENT LINE COUNT
      M=M+1
C             IS PAGE FINISHED
      IF(M-50)101,101,108
C             YES START NEW PAGE WITH PROPER HEADINGS
  108 WRITE OUTPUT TAPE 3,20,TITLE
      WRITE OUTPUT TAPE 3,20,(TITLE1(JL,J),JL=1,12)
      WRITE OUTPUT TAPE 3,25
C             SET LINE COUNT TO 1 FOR NEW PAGE
      M=1
C             NO  GO ON TO NEXT POINT
```

```
  101 CONTINUE
C
C              RETURN TO MAIN PROGRAM
      RETURN
      END
```

```
*      LIST
*      LABEL
*      SYMBOL TABLE
       SUBROUTINE RELTIM

CTIME CALCULATION OF TIME FOR THE DASHES
C
C                DIMENSION STATEMENTS
       DIMENSION CAX(2),CAY(2),CBX(2),CBY(2),CCX(2),CCY(2),COSZ(250,2),CO
      1SZ1(5,2),CPHI(2),CPHIP(2),CTH(2),D(250,2),DLONG(2),EL(5,2),ELAZ(25
      20,2),ELC(2),ELETA(2),ELEV(2),ELEXI(2),ELON(2),ELONH(2),ELONM(2),EL
      3ONS(2),ELP(2),ELZ(2),EM(5,2),EMAZ(250,2),EMC(2),EMETA(2),EMEXI(2),
      4EMP(2),EMZ(2),EN(5,2),ENAZ(250,2),ENC(2),ENETA(2),ENP(2),ENZ(2),ET
      5AP(250,2),EXIP(250,2),F2(2),GTHR(2),GTM(2),GTS(2),H(5,2),HEIGHT(25
      60,2),IFEXT(2),IFPLUS(2),NDASH(250,2),NUM(2),NWT(250,2),OCCULT(2),P
      7(2),PEL(250,2),PEM(250,2),PEN(250,2),PHD(2),PHI(2),PHIP(2),PHM(2),
      8PHS(2),R(5,2),RANGE(250,2),RHO(2),SINL(2),SLOPE(2),SPHI(2),SPHIP(2
      9),STH(2),THETA(2),TIME(250,2),TITLE(12),TITLE1(12,2)
       DIMENSION X(5,2),XBAR(2),XC(2),XP(250,2),XQ(2),XTR(30),Y(5,2),YBAR
      1(2),YC(2),YP(250,2),YQ(2),YTR(30),ZETAP(250,2),ZTRAIL(2)
C
C                COMMON STATEMENTS
       COMMON CAX,CAY,CBX,CBY,CCX,CCY,CDL,COSZ,COSZ1,CPHI,CPHIP,CTH,D,DL,
      1DLONG,DTRAD,EL,ELAZ,ELC,ELETA,ELEV,ELEXI,ELON,ELONH,ELONM,ELONS,EL
      2P,ELR,ELZ,EM,EMAZ,EMC,EMETA,EMEXI,EMP,EMR,EMZ,EN,ENAZ,ENC,ENETA,EN
      3P,ENR,ENZ,ETAAB,ETAAB0,ETAP,EXIAB,EXIAB0,EXIP,F2,GTHR,GTM,GTS,H,HE
      4IGHT,IFEXT,IFPLUS,J,K,N,NDASH,NUM,NWT,OCCULT,P,PEL,PEM,PEN,PHD,PHI
      5,PHIP,PHM,PHS,PI,R,RANGE,RHO,SDL,SINL,SINQ,SLOPE,SPHI,SPHIP,STH,TC
      6ORR,THETA,THETOH,THETOM,THETOS,TIME,TITLE,TITLE1,X,XBAR,XC,XP,XQ,X
      7TR,X0,Y,YBAR,YC,YP,YQ,YTR,Y0,ZETAAB,ZETAP,ZTRAIL
C
C                FORMAT STATEMENTS
   20 FORMAT(12A6)
   21 FORMAT(4H0XQ=1PE15.7,7H     YQ=1PE15.7,7H     XC=1PE15.7,7H     YC=1P
     1E15.7/4H F2=1PE15.7,7H      P=1PE15.7,11H     OCCULT=1PE15.7)
   22 FORMAT(127H0 DASH        X                TIME                RANGE
     1           HEIGHT               ALPHA                BETA
     2 GAMMA)
   23 FORMAT(1H I5,F10.3,1P6E19.7)
   24 FORMAT(22H0DIVIDE CHECK AT DY/DX)
   26 FORMAT(I5,I3,2F10.6,F9.3,F8.3,3F9.6)
C
C                USE LONG FORM WHEN PROJECTION AND SHUTTER ROTATION
C                CENTERS MORE THAN A CM APART
C                LONG FORM FOR CALCULATING OMEGA--EQ. 44
       OMEGF(XMEAS)=ATANF(Z*(W*(XMEAS-XQ1)-DYTR)/(W*DYTR+(XMEAS-XQ1)))
C                SHORT FORM FOR COMPUTING OMEGA---EQ. 42
       SOMEGF(XMEAS)=ATANF((XMEAS-XQ1)/DYTR)
C                COMPUTE DT BY EQ. 43
       DTIF(OMEGA)=0.15915494*SIGN*(OMEGA-OMEGA0)*PER
C
C                CALCULATE CONSTANTS
       CON=P(J)/OCCULT(J)
       TWOPI=2.*PI
C
       MUM=NUM(J)
C                REDEFINITIONS OF QUANTITIES TO USE WITH FUNCTIONS
       XQ1=XQ(J)
```

```
      PER=P(J)
C              DISTANCE BETWEEN CENTER OF ROTATION AND PROJECTION
      DX=XQ(J)-XC(J)
      DY=YQ(J)-YC(J)
      W=DY/DX
C
C              MAY GIVE DIVIDE CHECK IF DX VERY SMALL--NEED NOT AFFECT
C              RESULTS IF SOMEGF USED
      IF DIVIDE CHECK  6,7
    6 WRITE OUTPUT TAPE 3,24
    7 D2=DX**2+DY**2
      Z=SQRTF(1.+D2/F2(J))
C
C
C              CALCULATION OF OMEGAO
C              DISTANCE OF POINT FROM Y OF ROTATION CENTER
      DYTR=YP(1,J)-YQ(J)
C              X COORDINATE OF THE FIRST POINT
      XMEAS=XP(1,J)
C              NEED WE USE THE LONG FORMULA
      IF(D2-100.)10,10,11
C              NO SET UP TO USE SHORT FORMULA
   10 ASSIGN 50 TO KO
      ASSIGN 51 TO MO
      GO TO 70
C              YES SET UP TO USE LONG FORMULA
   11 ASSIGN 60 TO KO
      ASSIGN 61 TO MO
   70 GO TO KO,(50,60)
C              COMPUTE OMEGAO BY SHORT FORMULA---EQ. 42
   50 OMEGAO=SOMEGF(XMEAS)
C              SET CODE FOR SHORT FORMULA USE
      AA1=1.
      GO TO 71
C              COMPUTE OMEGAO BY LONG FORMULA---EQ. 44
   60 OMEGAO=OMEGF(XMEAS)
C              SET CODE FOR LONG FORMULA USE
      AA1=2.
C
C              LOOP FOR EACH POINT
C              J=NUMBER OF STATION---K=NUMBER OF POINT AT STATION
   71 DO 100 K=1,MUM
C              COMPUTE DIRECTION COSINES OF POINT IN ALTITUDE AZIMUTH
C              SYSTEM---EQ. 12
      ELAZ(K,J)=PEM(K,J)*CTH(J)-PEL(K,J)*STH(J)
      EMAZ(K,J)=PEN(K,J)*CPHI(J)-(PEL(K,J)*CTH(J)+PEM(K,J)*STH(J))*SPHI(
     1J)
      ENAZ(K,J)=(PEL(K,J)*CTH(J)+PEM(K,J)*STH(J))*CPHI(J)+PEN(K,J)*SPHI(
     1J)
C              X MEASUREMENT OF POINT
      XMEAS=XP(K,J)
C              DISTANCE OF Y TO PROJECTION CENTER
      DYTR=YP(K,J)-YQ(J)
C              COMPUTE OMEGA OF POINT BY PROPER FORMULA
      GO TO MO,(51,61)
   51 OMEGA=SOMEGF(XMEAS)
      AA2=1.
      GO TO 72
```

```
   61 OMEGA=OMEGF(XMEAS)
      AA2=2.
C             SIGN OF ROTATION DIRECTION
   72 SIGN=IFPLUS(J)
C             COMPUTE DT FOR POINT---EQ. 43
      DT=DTIF(OMEGA)
C             DASH NUMBER
   75 ENDASH=NDASH(K,J)
C             COMPUTE RELATIVE TIME OF POINT---EQ. 41
  100 TIME(K,J)=CON*ENDASH+DT
C
C             WRITE OUT TITLE AND CAMERA PARAMETERS AND HEADINGS
      WRITE OUTPUT TAPE 3,20,TITLE
      WRITE OUTPUT TAPE 3,20,(TITLE1(JL,J),JL=1,12)
      WRITE OUTPUT TAPE3,21,XQ(J),YQ(J),XC(J),YC(J),F2(J),P(J),OCCULT(J)
  103 WRITE OUTPUT TAPE 3,22
C             SET LINE COUNT TO 1 AT START OF PAGE
      M=1
C
C             WRITE OUT DASH NUMBER,X READING,RELATIVE TIME,RANGE,HEIGHT
C             AND DIRECTION COSINES ALTITUDE AZIMUTH SYSTEM EACH POINT
      DO 104 K=1,MUM
      WRITE OUTPUT TAPE 3,23,NDASH(K,J),XP(K,J),TIME(K,J),RANGE(K,J),HEI
     1GHT(K,J),ELAZ(K,J),EMAZ(K,J),ENAZ(K,J)
C             INCREMENT LINE COUNT
      M=M+1
C             END OF THIS PAGE
      IF(M-50)104,104,106
C             YES SET UP NEXT PAGE
  106 WRITE OUTPUT TAPE 3,20,TITLE
      WRITE OUTPUT TAPE 3,20,(TITLE1(JL,J),JL=1,12)
      WRITE OUTPUT TAPE 3,22
C             SET LINE COUNT TO 1
      M=1
C             NO GO ON TO NEXT POINT
  104 CONTINUE
C
C             DO WE WANT CARDS
      IF(IFEXT(J))200,201,200
C             YES WRITE DISTANCE,TIME,RANGE,HEIGHT,DIRECTION COSINES
C             ,DASH NUMBER AND DASH WEIGHT FOR EACH POINT ON CARDS
  200 WRITE OUTPUT TAPE 14,20,TITLE
      WRITE OUTPUT TAPE 14,20,(TITLE1(JL,J),JL=1,12)
      DO 204 K=1,MUM
      WRITE OUTPUT TAPE 14,26,NDASH(K,J),NWT(K,J),D(K,J),TIME(K,J),RANGE
     1(K,J),HEIGHT(K,J),ELAZ(K,J),EMAZ(K,J),ENAZ(K,J)
  204 CONTINUE
C             RETURN TO MAIN PROGRAM
  201 RETURN
      END
```

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13. ABSTRACT

The Optical Reduction Programs have been written to obtain the space trajectory of a re-entering body from measurements of position on two photographic plates taken of a re-entry event from two separate stations. The method of analysis follows closely that developed by Whipple and Jacchia. The Programs are a modification for the Lincoln Laboratory 7094 computer of a program originally written at the Harvard Observatory for reduction of meteor trails. The computation is divided into two programs: (1) The Plate Calibration Program calibrates the plate by using coordinates and plate measurements of known stars as reference points. (2) The Optical Trajectory Program computes range, height, time, direction cosines, and distance along the trail from the measured points on the trail. This report discusses these two programs in detail, including program listings, flow charts, and directions for running the program. The mathematical background and the experimental method for obtaining the input data are also discussed.

14. KEY WORDS

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Trailblazer I
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